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Wind Flow Induced Vibrations of Tapered Masts

Ahmad Bani-Hani
Cleveland State University

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AHMAD BANI-HANI

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The University of Jordan

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This thesis has been approved

for the Department of **MECHANICAL ENGINEERING**

and the College of Graduate Studies by

Thesis Committee Chairperson, Dr. Majid Rashidi

Department & Date

Dr. Rama S.R.Gorla

Department & Date

Dr. Earnest N. Poulos

Department & Date

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ABSTRACT

Structural dynamic analyses of elongated masts subjected to various wind speeds are presented in this work. The masts are modeled as vertically supported cantilever beams, with one end fixed to the ground, and the other end free. The external excitation forces acting of the masts are the results of vortex shedding represented by a sinusoidal time dependent functions. The frequencies of these sinusoidal functions are dictated by the Strouhal numbers associated of the flow regimes crossing over the masts. To enhance the vibratory behavior of a typical mast, under the influence of flow induced vibrations, three different mass distributions along the length of the mast were considered. The different mass distributions were achieved by tapering the mast along its length, allocating more of the mass at its fixed end, and gradually decreasing it toward its free end. Three different tapering angles were considered for these studies. All results were compared with the results obtained for a straight circular cross-section cylindrical mast having the same overall length and mass. For a mast length of 25 meters and a total mass of 1782.74 kg, three tapered angles of 0.229, 0.458 , and 0.596 degrees, were considered. These analyses show that the first natural frequencies of the tapered masts increases from that of the straight mast. The first natural frequency of the straight mass was determined to be 0.2845 Hrz. The corresponding first natural frequencies for the masts with tapered angle of 0.229, 0.458, and 0.596 became 0.417, 0.5911, and 0.7435Hrz. respectively. In addition to the natural frequency analyses, dynamic

responses analyses of these masts were determined under the influence of the harmonic excitations resulting from the vortex shedding cause by the wind flow. For the tapered angles chosen in these studies the maximum displacement of the free ends of these masts were determined. For a wind speed of 10 m/s the free-end displacements of the tapered masts were determined to be (500, 1000, 1300)* 10^{-6} meters for tapered angles of 0.229, 0.458, 0.596 degrees respectively. The computational analyses performed in this work were accomplished by using SAP 2000 software.

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NOMENCLATURE

A	cross sectional area of the mast, or of the frame element used in SAP2000, m²
A_p	projected area of a circular mast perpendicular to the direction of flowing fluid (air), m²
A_{s2}	shear area of a frame element section, inSAP2000, for transverse shear in the 1-2 plane, m²
A_{s3}	shear area of a frame element section, in SAP2000, for transverse shear in the 1-3 plane, m²
a	outside radius of the tapered mast at the fixed base, m
b	outside radius of the tapered mast of the free end, m
C	constant representing the amplitude of the straight mast sinusoidal mode shape or normal function.
C_1, C_2, C_3, C_4	constants representing the components of the amplitude of the straight mast sinusoidal mode shape or normal function.
$\bar{C}$	dimensionless correction factor ($\bar{C} = 2$ for tubular cross section).
C_d	fluctuating drag coefficient.
C_l	fluctuating lift coefficient.
D	outside diameter of a straight mast, m
d	inside diameter of straight mast, m
$\bar{D}$	average of outside diameters of both fixed and free ends of a straight or

tapered mast, m

E Young's modulus of elasticity, N/m²

E_{AL} Aluminum modulus of elasticity, N/m²

f cyclic frequency of a mode of vibration (Hz)

f_k k^{th} iteration of the cyclic frequency f a model of vibration.

F concentrated load acting at the tip of the straight or tapered mast in the positive x-direction, N

$\bar{f}$ external distributed force per unit length of the mast, N/m

$\check{f}$ constant time function, along the length of the mast, of the vector of applied loads

$\tilde{f_\iota}$ i^{th} time function, along the length of the mast, of the vector of applied loads

$\check{f}_s$ constant time function, along the length of the mast, of the s^{th}-vector of applied loads,

F_l harmonically varying lift force of vortex shedding in the cross-wind direction, N

F_{sl} s^{th}harmonically varying lift force of vortex shedding in the cross wind direction, N

f_s frequency of vortex shedding, Hz

F_d harmonically varying drag force of vortex shedding in the along wind direction, N

f_m frequency of mast's oscillation, rad/s

G shear modulus or modulus of rigidity, N/m^2

G_{AL}	Aluminum modulus of rigidity
h	positive constant equals square the natural frequency of vibration of the straight mast,
I	rectangular second moment of the mast cross sectional area about the neutral axis (y-axis), m^4
I_{22}	rectangular second moment of area about the neutral axis, local axis 2, of the frame element section in SAP2000, m^4
I_{33}	rectangular second moment of area about the neutral axis, local axis 3, of the frame element section in SAP2000, m^4
J	polar second moment of the mast's cross sectional area, or torsional constant of the frame element section in SAP2000, m^4
K	structural stiffness matrix
L	height of the straight or tapered mast, m.
M	bending moment, $\mathrm{N.m}$.
$\bar{M}$	diagonal mass matrix.
N	axial force sousing tension or compression of the mast, N.
O	proportional damping matrix.
P_i	i^{th} spatial load vector of the vector of applied loads, N
P_0	spatial load vector at base of the mast of the vector of applied loads, N
P_{20}	20^{th} spatial load vector at the free end of the mast of the vector of applied loads, N
r	vector of applied loads
r_s	s^{th} vector of applied loads

$r_i(z)$	inside radius of the tapered mast, taken at any point, in the z-direction; along the mast's longitudinal axis.
$r_o(z)$	outside radius of the tapered mast, taken at any point, in the z-direction; along the mast's longitudinal axis.
Re	Reynolds number.
s	Complex argument of exponential mode shape or normal function
s_1, s_2, s_3, s_4	constants representing the roots of the auxiliary equation.
St	Strouhal number
t	time, s
$\bar{t}$	wall thickness of straight or tapered mast,m
$\bar{T}$	torque, putting the mast under torsion, N-m
$T(t)$	a function depending only on time, and represents the variation of mast's displacement with respect to time under separation of variables method.
$\tilde{T}$	cyclic period of a mode of vibration.
$\tilde{T}_k$	k^{th}iteration of the cyclic period of a mode of vibration
U	velocity of wind or flowing air, m/s
U_{cr}	critical wind speed, m/s
U_r	reduced flow velocity ,m/s
U_s	s^{th} velocity of flowing air,m/s
u	vector of relative displacements with respect to the ground
$\dot{u}$	vector of relative velocities with respect to the ground
$\ddot{u}(t)$	vector relative accelerations with respect to the ground
$\bar{U}_1$	strain energy due to tension or compression, J

$\overline{U}_2$	strain energy due to shear,
$\overline{U}_3$	strain energy due to torsion
$\overline{U}_4$	strain energy due to bending moment
$\overline{U}_t$	total strain energy of the mast
V_{st}	volume of the straight mast, m^3
V_{t1}	volume of the outer frustum of the tapered mast, m^3
V_{t2}	volume of the inner frustum of the tapered mast, m^3
V_{tap}	volume of the tapered mast, m^3
V	shear force, N
w	transverse displacements of the straight mast.
$W(z)$	a function depending only on the z-coordinate value, and represents the variation of masts displacement in the z-direction along its longitudinal axis.
x	x-coordinate of the Cartesian coordinate system
y	y-coordinate of the Cartesian coordinate system
z	z-coordinate of the Cartesian coordinate system
z_i	location of i^{th} spatial load vector in the z-direction; along the mast longitudinal axis.

OTHER SYMBOLS

ρ	mass density, $\mathrm{kg/m^3}$
ρ_{air}	mass density of flowing air, $\mathrm{kg/m^3}$
ρ_{AL}	mass density of Aluminum, $\mathrm{kg/m^3}$
ρ_w	weight density of anisotropic material assigned to a prismatic section in SAP2000, $\mathrm{N/m^3}$
μ	dynamic or absolute viscosity , $\mathrm{N.s/m^3}$
$\bar{\mu}$	Eigenvalue relative to the frequency shift.
$\bar{\mu}_k$	k^{th} iteration of the Eigenvalue relative to the frequency shift.
ν	Kinematic viscosity, $\mathrm{m^2/s}$
ν_{air}	Kinematic viscosity of flowing air, $\mathrm{m^2/s}$
γ	coefficient of thermal expansion, $\mathrm{ppm/C^o}$
δ	static deflection in the direction of the force, m
$\delta_1, \delta_2, \delta_3, \delta_4$	static deflections of tip of tapered masts I, II, and III respectively in the direction of the force, m
δ_{str}	static deflection of the tip of the straight mast in the direction of the force, m
α	constant equals $\sqrt{EI/\rho A}$
β	argument of the beam sinusoidal mode shape or normal function.
β_n	argument of the beam n^{th} sinusoidal mode shape or normal function
Ω^2	diagonal matrix of Eigenvalues
ϕ	matrix of Eigenvector (mode shapes).

ω natural frequency of the mast, rad/s

ω_n n^{th} natural frequency of the mast, rad/s

$\bar{\omega}$ circular frequency of vortex shedding, rad/s

$\bar{\omega}_s$ circular frequency of vortex shedding of the constant time function of the s^{th} vector of applied load, rad/s

ω_0 equals frequency shift multiplied by 2π

τ_s period of the constant time function of the s^{th}vector at applied loads, s operations.

SUBSCRIPTS

n number of the mode shape or natural frequency.

s number of the vector of applied loads, or velocity of flowing air.

i number of the time function, or spatial load vector.

k iteration number of the cyclic frequency or period of a mode of vibration

OPERATIONS

dv Elemental change, in shear.

dz Element length.

dM Elemental change in moment

$. = \frac{d}{dt}$ First time derivate

$\ddot{} = \frac{d^2}{dt^2}$	Second time derivative
$\frac{\partial^2}{\partial t^2}$	Second partial derivative with respect to time
$\frac{\partial}{\partial z}$	First partial derivative with respect to z.
$\frac{\partial^2}{\partial z^2}$	Second partial derivative with respect to z
$\frac{\partial^4}{\partial z^4}$	Fourth partial derivative with respect to z.
$\frac{d}{dz}$	First derivative with respect to z
$\frac{d^2}{dz^2}$	Second derivative with respect to z
$\frac{d^3}{dz^3}$	Third derivative with respect to z
$\frac{d^4}{dz^4}$	Fourth derivative with respect to z

CHAPTER I

INTRODUCTION TO FLOW INDUCED VIBRATION AND VORTEX SHEDDING PHENOMENON

1.1 Introduction and background

Masts are commonly used in varieties of application. They are used to raise flags, support light fixtures on the highways, support lightning rods for protection of building, etc. In most applications they are under wind loads which make the studying of their dynamic response under these conditions essential.

Other examples of use for masts are: radio and television broad casting, telecommunication, two way radio (emergency response systems such as police and fire), and paging systems require elevated antennas, which may be supported on tall masts. The increasing reliance of businesses on instant voice and data communication by telephone, the considerable money involved in broadcasting by a mature, omnipresent radio and television industry, and the increased reliance on radio and telephone telecommunication for emergency systems, requires an increased sensitivity to reliability of those masts. This is especially true in conditions likely to introduce significant dynamic response as in the case of wind storms [1].

Another application is the vibration of a yacht mast due to wind which causes noise below decks since the hull acts like a sounding board and amplifies the vibration noise.

Tall slender steel chimneys in industrial plants and the affect of flow induced vibration on their stability is another example of mast vibration. The departments of transportation use long tapered steel poles for closed circuit television (CCTV) cameras the image of which is transmitted for traffic monitoring. Traffic signal poles and their dynamic behaviors under wind force is another example of mast vibration.

Highway lighting poles are slender structures usually characterized by low values of structural damping; a factor that can lead to large amplitude vibrations when excited by a varying wind force.

The above mentioned applications are examples from a long list which emphasize the importance of studying the dynamic behavior and stability of masts under varying wind loads.

This study addresses the vibration of a typical vertically supported mast made from Aluminum under various wind loads. The dynamic behavior of the mast in this study falls under the category of flow induced vibrations of structures. This flow- induced vibration will be analyzed by finite element method in order to determine the critical wind speed that could potentially create resonance phenomena for the vibration mast under the action of the wind. This critical condition occurs when one of the natural frequencies of the mast coincides with the frequency of the vortices shedding around the mast as the result of being subjected to the wind. Under these conditions the mast undergoes dangerously large oscillation which may lead to structural instability and/or structural failure. In order to improve the dynamic behavior of the mast, its shape and mass distribution could be modified from that of a straight cylinder. In other words, by re-distribution of the mass along the length of the mast its natural frequencies could be affected. To modify the mast, its natural frequencies have to increase to higher values.

1.2 Flow induced vibration of a mast

Flow induced vibration is a term to denote those phenomena, associated with response of structure immersed in or conveying fluid flow. The term covers those cases in which an interaction develops between fluid dynamic forces and the inertia, damping or elastic forces in the structures. The study of these phenomena draws on three disciplines, structural mechanics, mechanical vibration, and fluid dynamics.

If a cylinder is immersed in a moving fluid, such as a mast subject to wind, the moving air (wind) exerts dynamic forces onto the surface of the cylinder. There are two types of forces acting on the cylinder in this case; these are the pressure drag and the viscous forces. The pressure drag forces are always perpendicular to the surface, and the viscous forces are parallel to the surface of the cylinder. The sum of these forces (viscous, pressure, or both) that acts normal to the free-stream direction is the lift, and the sum that acts parallel to the free-stream direction is the drag. The resultant effect of the lift and drag forces is called the fluid excitation forces [2]. Also because of cylinder displacement in a flow of air, the cylinder will be subjected to a fluid force that is proportional to the cylinder displacement called fluid stiffness force [3].

When a body (cylinder) moves at a variable velocity in a viscous flow, it experiences resistance; the cylinder behaves as though an added mass of fluid were rigidly attached to and moving with it, this force is called fluid inertial force. Due to the viscosity there is phase difference between cylinder acceleration and fluid acceleration, which results in another force opposing the movement of the cylinder called fluid damping force. In general all the previous forces have coefficients in their equations that depend on cylinder displacement, velocity, and acceleration in addition to the flow velocity; these fluid forces are non linear functions of cylinder motion [3].

The type of response of the structure (cylinder) depends on the excitation mechanism. The main concern in this study is the dynamic response and stability of a cylinder, subjected to periodic excitation represented by vortex shedding.

1.3 Vortex Shedding and Strouhal number

When a flow of air [2] (viscous real fluid) past a circular cylinder, and if we visualize a fluid particle as it travels around the cylinder from A to B to C and finally to D, Figure 1.1, we first see that it decelerates, consistent with the increase in pressure from A to B (stagnation point). Then as it passes from B to C, the stream lines converge causing the velocity to increase and the pressure in the fluid to decrease between B and C.

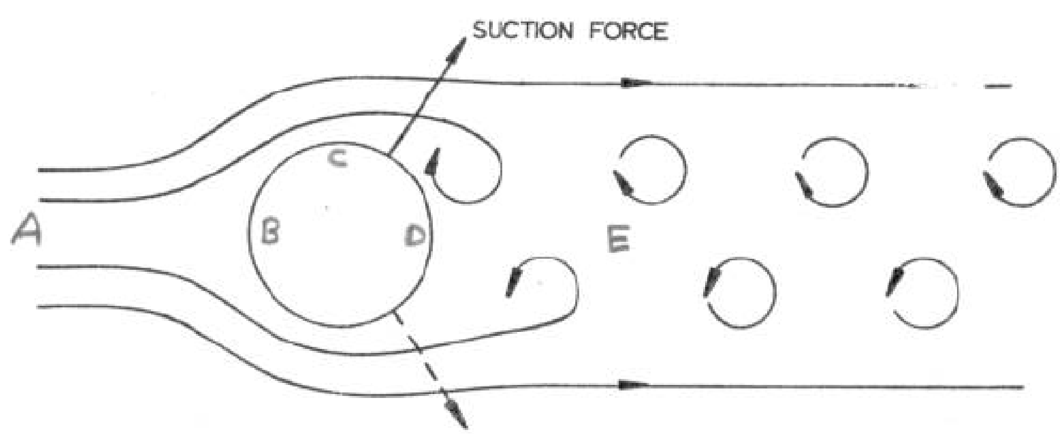

Figure 1.1 Pattern of flow of a real fluid passed a circular cylinder

In addition to that and very close to the cylinder surface between B and C a thin layer of fluid has its velocity reduced from that of the free stream due to viscous resistance.

Actually, the fluid particles directly adjacent to the surface have zero velocity. This reduced velocity layer is called the boundary layer, which has a tendency to grow in thickness in the direction of flow. However, because the main stream of fluid outside the boundary layer is accelerating in the same direction, the boundary layer remains quite thin up to approximately the midsection. Downstream of the midsection (from C to D) the streamlines begin to diverge leading to a reduction in velocity with an attendant increase in pressure. For viscous flow deceleration of the fluid next to the boundary is limited because its velocity is already small due to the viscous resistance. Therefore, the fluid near the boundary can proceed only a very short distance against the adverse pressure gradient before stopping completely. When the motion of the fluid next to the boundary cease, this causes the main stream of the flow to be diverted a way or to be separated from the boundary. Downstream of the point of separation the fluid outside the surface of separation has a high velocity and the fluid inside the surface of separation has a relatively low velocity. Because of the steep velocity gradient along the surface of separation, eddies are generated and then detached at fairly regular intervals and move downstream to form a large wake. It is a rule of thumb that the pressure that prevails at the point of separation also prevails over the body within the zone of separation. Consequently pressure on rear half of the cylinder is much less than the pressure on the front half [2].

The location of the point of separation on the cylinder depends on the character of the flow in the boundary layer (laminar or turbulent). When the boundary layer is laminar separation tends to occur further up stream than when it is turbulent. This is due to the fact that in laminar flow, the velocity near the surface is lower and much less friction force is required to produce reversal of flow. The more vigorous transverse mixing of fluid particles which occurs in turbulent flow discourages flow reversal near the surface and tends to delay separation. The position of the separation point therefore moves downstream with increasing boundary layer turbulence and this may occur as a result of increasing the turbulence in the approaching flow, increasing the roughness of cylinder surface, or by increasing Reynolds number (caused by higher velocity or greater body size). Since separation is closely associated with the viscous resistance of the fluid, it is not surprising that the Reynolds number, the value of which is inversely proportional to the relative viscous resistances, as shown in Equation (1.1), is an indicator of the onset of separation.

$$\boldsymbol{Re} = \frac{\rho U \bar{D}}{\mu} = \frac{U \bar{D}}{\nu} \qquad \textbf{(1.1)}$$

Where, $\boldsymbol{Re}$ is Reynolds number.

$\boldsymbol{\rho}$ is density of the flowing fluid (kg/m^3).

$\boldsymbol{U}$ is velocity of flowing fluid (m/s).

$\bar{\boldsymbol{D}}$ is the average of the outside diameters of the fixed and free ends of the straight or tapered mast (m).

$\boldsymbol{\mu}$ is dynamic or absolute viscosity (N.s/m^2).

$\mathbf{\nu}$ is the kinematic viscosity (m^2/s).

The use of the average of outside diameters, $\overline{D}$, was proposed for simplicity. It also simplifies the calculation of the projected area of the mast perpendicular to the direction of airflow.

For a circular cross-section mast, the precise nature of the wake depends on the value of the Reynolds number, as shown in Figure 1.2 below.

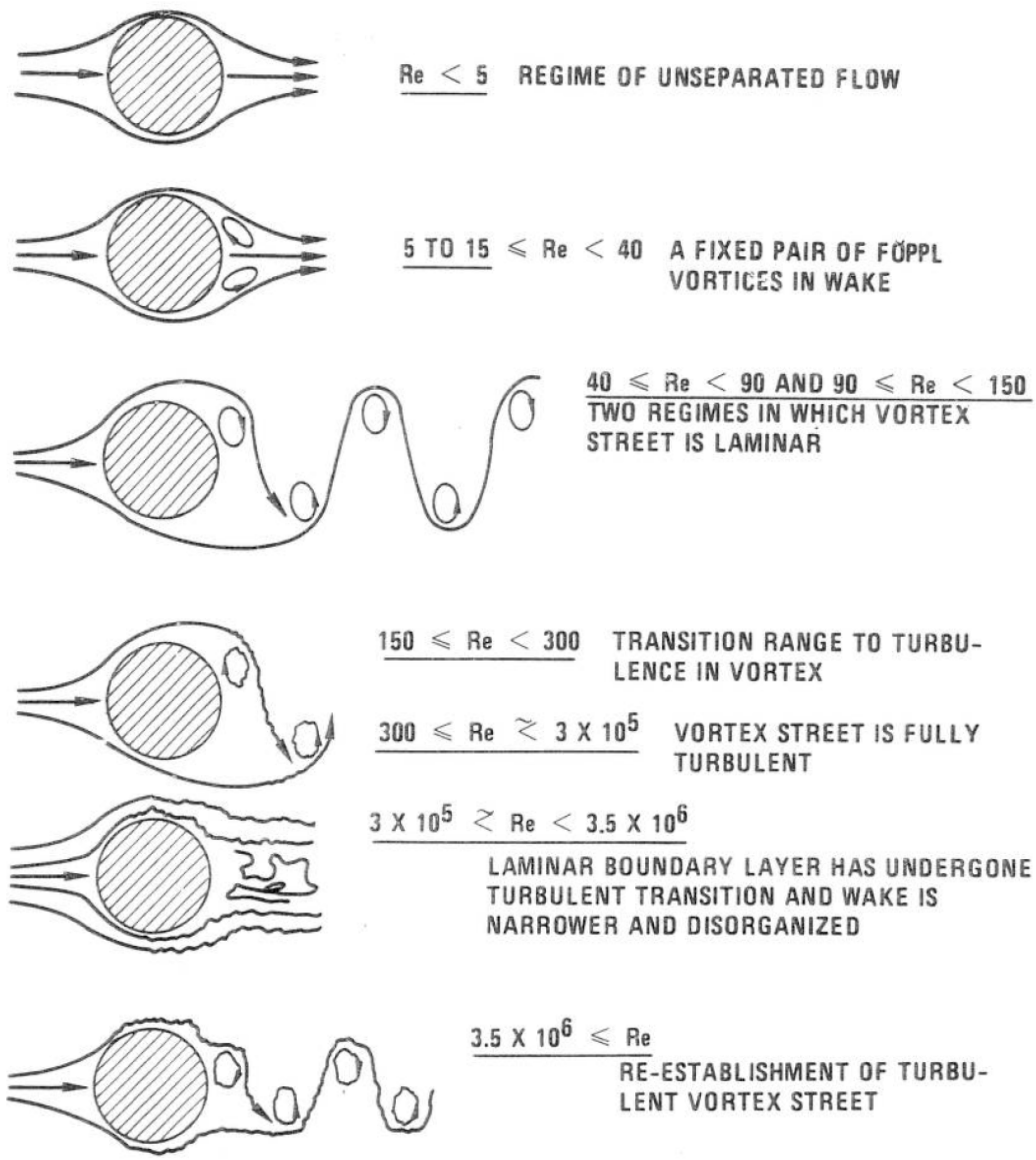

Figure 1.2 Regimes of fluid flow across circular cylinders (Reference: Flow-Induced Vibrations of circular cylindrical structures, by: Shoei-Sheng Chen, pages 249)

At very low Reynolds number ($Re < 5$) the flow does not separate. As Re is increased, a pair of fixed vortices is formed immediately behind the cylinder (5 to 15 < Re < 40). As Re is further increased, the vortices elongate until one of the vortices breaks away and a periodic wake and staggered vortex street is formed. In the range (40< Re <150) the vortex street is laminar, and in the range (150 < Re <300) the vortex street is in the transition from laminar to turbulent. At a Re of 300, the vortex street is fully turbulent. The Re range of 40 to approximately $3 * 10^5$ has been called the sub critical range. In this range the boundary layer is laminar and separation occurs up stream of the point of lowest surface pressure, causing a large wake with large drag coefficient. The pattern of vortex shedding in this range is regular (shedding occurs at a well defined frequency) [3].

The onset of separation point instability (between forward and downward of the center section) occurs at the "critical Reynolds number"; approximately $3 * 10^5$. Its precise value depends on the roughness of surface and on the amount of turbulence in the main stream. The separation point instability occurs over a range of Reynolds values ($3 * 10^5 < Re < 3.5 * 10^6$) called the transition range. In the transition range the wake is confused with varying width and randomly distributed eddies, which leads to a disorganized vortex shedding with broadband shedding frequencies. At $Re > 3.5 * 10^6$ (the super critical range) the boundary layer is turbulent, the location of the points of separation is stable and downward of the center section leading to narrow wake and the pattern of vortex shedding once again regular [3].

During the regular patterns of vortex shedding [5], the frequency of vortex shedding f_s (in units of Hertz) from a circular mast in a uniform flow of air is related to the mast average outside diameter ($\overline{D}$) and the flow velocity (U) through the non dimensional Strouhal number St , as illustrated in Equation (1.2)

$$\boldsymbol{St = \frac{f_s \bar{D}}{U}} \qquad \textbf{(1.2)}$$

Again $\bar{D}$ is assumed, to simplify future calculations.

If the cylinder is inclined toward the flow, the component of flow velocity normal to the cylinder axis is used in the above equation. Strouhal number is a function of geometry and Reynolds number. For sharp edged bluff bodies in which the point of boundary layer separation is independent of Re number, the Strouhal is invariant with the Reynolds number, but depends on body shape, most sharp edged shapes have approximately (0.15) value for St. For rounded shapes, circular cross section mast, the Strouhal number depends on Reynolds number. Figure1.3 shows the relationship between Re and St for a circular cylinder [6].

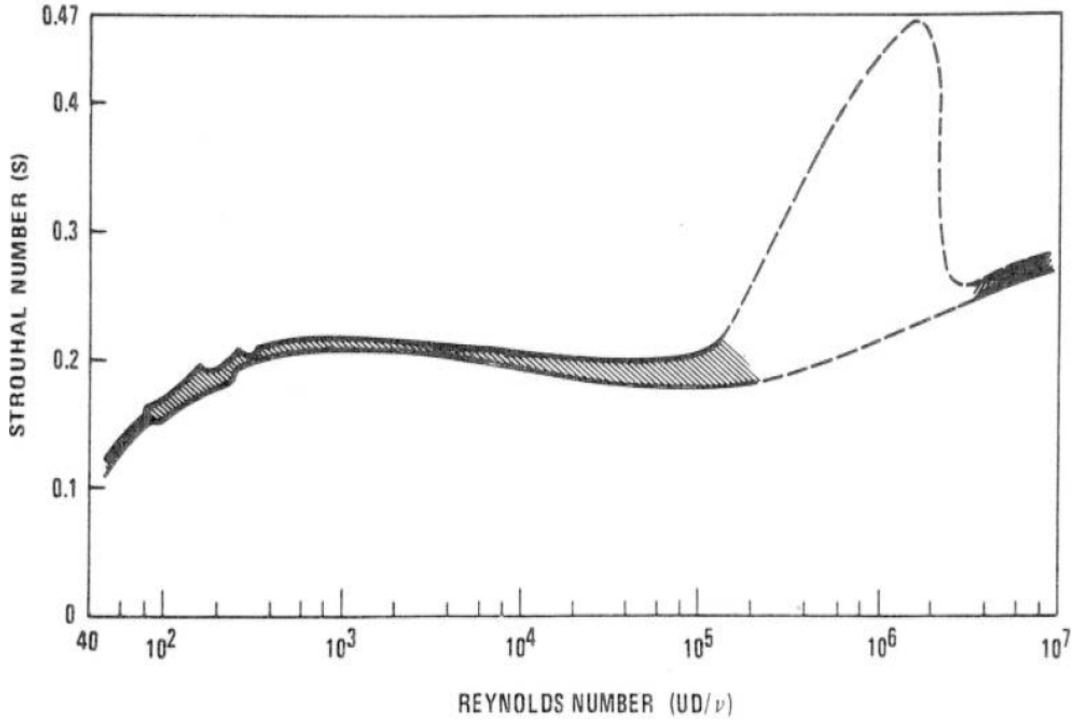

Figure 1.3 The Strouhal-Reynolds number relationship for circular cylinders
(Reference: Flow induced vibrations, by: Robert D.Belvins,Page:15)

The Strouhal number remains nearly constant with a value of (0.21) in the range of Reynolds numbers (300- $2 * 10^5$). In the range of Re numbers from $2 * 10^5$ to $3.5 * 10^6$ (transition range), St number increases depending on the turbulence intensity of the incoming flow, and the shedding frequency is defined in terms of the dominate frequency of a broadband of shedding frequencies. Above Reynolds number of $3.5 * 10^6$, Strouhal number remains constant ($St \approx 0.27$) [5].

1.4 Periodic excitation of the mast by vortex shedding

In the stable ranges, where the point of separation is stationary, the regular pattern of eddy shedding gives rise to a variation in pressure a cross the leeward side of the mast which has a regular periodicity and which may give rise to a dynamic response. The vortices which form regular wake patterns are normally of approximately equal intensity, have opposite circulation velocities and detach themselves alternately from either side of the mast to form what is called Vortex Street. This causes suction forces of periodic nature with two components; cross-wind component and along wind component. The mast is therefore is subjected to a large periodic cross-wind force which has a frequency equal to that of the vortex shedding frequency, and a small along-wind periodic force with a frequency equal to twice vortex shedding frequency. If the natural frequency of the mast corresponds with either of these frequencies and oscillatory response may result. Practically, the forces in the along-wind direction are so small that a vibration at this frequency is unusual and the oscillatory response in the cross-wind direction which has larger value of forces is the one of concern and investigation [4].

For the range of Reynolds numbers from 300 to $2 * 10^5$, where the value of Strouhal number is approximately equal 0.21, yields

$$\mathbf{0.21} = \frac{f_s \bar{D}}{U} \qquad \textbf{(1.3)}$$

The harmonically varying lift force $F_l\,(t)$ is given by [3]

$$F_l(t) = \frac{1}{2}\, C_l\, \rho_{Air} U^2 A_p \;\; \sin \bar{\omega} t \qquad \textbf{(1.4)}$$

Where, $\boldsymbol{C_l}$ is the fluctuating lift coefficient ($C_l \approx 1$ for a circular cylinder).

$\boldsymbol{\rho_{air}}$ is the density of the flowing air (Kg/m^3).

$\boldsymbol{U}$ is the velocity of the flowing air (m/s).

$\boldsymbol{A_p}$ is the projected area of the mast perpendicular to the direction of U (m^2).

$\boldsymbol{\overline{\omega}}$ is the circular frequency of vortex shedding (rad/s), $\overline{\omega} = 2\pi f_s$.

Where $\boldsymbol{f_s}$ is the frequency of vortex shedding in Hertz

$\boldsymbol{t}$ is time (s).

For the harmonically varying drag force , $F_d(t)$, Equation (1.4) can be used by simply replacing (C_l) by (C_d) which is the fluctuating drag coefficient, and changing ($\overline{\omega}$) into ($2\overline{\omega}$). The value of the fluctuating drag and lift coefficients is a function of Reynolds number, turbulence characteristics of incoming flow, surface characteristics (roughness), and oscillation amplitude. The use of (1) for C_l and (0.2) for C_d is considered conservative for all Reynolds numbers [3].

The characteristics of vortex shedding and the response of the mast can be studied or analyzed with respect to the reduced flow velocity , U_r, which represents the ratio of fluid kinetic energy to strain energy in the structure, and expressed mathematically as [3]

$$\boldsymbol{U_r = \frac{U}{f_m D}} \qquad \textbf{(1.5)}$$

Where, $\boldsymbol{f_m}$ is the frequency of mast's oscillations (rad/s).

$\boldsymbol{U}$ is the velocity of the flowing air (m/s).

$\boldsymbol{D}$ is the outside diameter of the straight mast (m).

Typically, when the natural frequency of the mast is considerably greater than the vortex shedding frequency, at low values of U_r , turbulence in the flow excites the mast to very small amplitude oscillations in both lift and drag directions at the natural frequency of the mast. Occasionally, the mast may also be excited by vortex shedding; the response is in the lift direction and its frequency is the vortex shedding frequency. This is called the zone of constant Strouhal number (St = 0.21) and increasing forcing frequency.

As the flow velocity, U_r , is slowly increased, the shedding frequency increases and the mast starts performing steady state oscillations with increasing amplitude in the drag direction. This is also accompanied by small amplitude oscillations in the lift direction. The dominant frequencies in both the lift and drag direction are at the cylinder natural frequencies. Further increase in the flow velocity U_r leads to a condition in which the mast natural frequency equals twice the vortex shedding frequency. Under this condition the mast oscillates with large amplitude in the drag direction at the mast natural frequency, and the vibration in the drag direction controls the vortex shedding over a range of flow velocity determined by the system damping. In the flow velocity range the vortex shedding frequency locks-on to half the mast natural frequency. This response is called lock-in in the in-line direction. It is worth noting that over this range of velocity, U_r, the mast still responds with relatively increasing amplitude in the lift direction at the vortex shedding frequency. As the flow velocity, U_r, continue to increase the amplitude in the drag direction falls off until eventually control of the shedding frequency is lost. At this point the vortex shedding frequency shifts bock to the constant Strouhal zone $(St = 0.21)$, and continue to increase with increasing velocity. During this stage the response amplitude in the lift direction continues to increase, and with a frequency equals the vortex shedding frequency.

When the vortex shedding frequency is close or equal to the mast natural frequency, vortex-exited oscillations in the lift direction becomes dominant. At this point, the vibration of the mast in the lift direction controls the vortex shedding frequency over a range of flow velocity, U_r , determined by the system damping. In this flow velocity range the vortex shedding frequency lock-on to the mast natural frequency. This response is called lock-in in the lift direction. In this range of flow velocity maximum amplitude in the lift direction is reached, at which point the input energy from the flow just balances that absorbed by the cylinder. A further increase in flow velocity causes the amplitude to fall off until eventually control of the shedding frequency is lost. At this point, if the natural frequency of higher modes and the damping is sufficiently high, the cylinder response amplitude becomes small. There is a possibility that two or more modes overlap over a small range of flow velocity U_r[3].

1.5 Work done to reduce flow induced vibration of masts

To control and reduce the flow induced vibration of mast a lot of devices and methods have been used, the majority of them fall under two major groups [3]:

a. Spoilers: which tend to change fluid dynamic characteristics of structures in such a way as to interfere with and weaken the exciting force resulting from vortex shedding. Some examples for spoilers are helical strakes, shrouds, slates, fairing, splitter plate, and flags.
b. Dampers: which provide a mechanism for dissipation of energy and that leads to an increase in the structural damping of the mast and thus reducing the amplitudes of forced vibration resulting from vortex shedding and finally reducing the possibility of structural damage or failure. Some examples for dampers are tune mass damper, tune liquid damper, impact damper.

Tune mass dampers (TMD) offer a relatively simple and effective way of reducing excessive vibrations in mast under flow induced vibration. By attaching a secondary mass to the mast with approximately the same natural frequency, large relative displacements between the mast and the secondary mass will occurs at resonance, and the mechanical energy of the system can then be dissipated by placing properly tuned damper between the two. The increase in the structural damping of the mast depends on the ratio of (TMD) mass to mast mass, tuning ratio which is the ratio of the frequency of (TMD) to that of the mast, and finally the (TMD) damping ratio [7].

Hanging chain impact damper (HCID) is an example of impact dampers used when the oscillation of mast due vortex shedding is essentially in one plane. (HCID) can be employed inside the mast, the impact of the chain against the internal wall of the mast, and the internal friction of chain links rubbing against each other provides two energy dissipation mechanisms which leads to an increase in the structural damping of the mast and reduction in the amplitudes of the forced vibration [8].

For tall masts guy cables between the top of the most and the ground are used. Attaching vibration dampers through guy cables will increase the structural damping of the mast, and thus reducing the amplitude of the flow induced vibration. By controlling the tension in guy cables, which is equivalent to stiffness variation of the mast, the amplitude of transverse structural vibration caused by varying wind forces will be minimized [9].

A single ball immersed in oil or viscous fluid in an upright cylindrical cup attached to the top of a mast represents a damper. For small amplitudes of vibration the relative motion between the ball and the cup will lead to viscous drag force acting on the ball which will dissipate energy, and for large amplitudes of vibration the ball will also impact against a cushion of oil on the sides of the cup. This dissipated energy will increase the mast structural damping and consequently reducing the amplitudes of fluid induced vibration. This kind of damper is more effective than chain damper and functions with a reduction in noise level [10].

Vortex induced vibration of a circular mast can be suppressed by acoustic excitation with the frequency of transition waves, the shear layers which separate from surface of a circular cylinder form periodic vortices behind the cylinder, which leads to vortex shedding phenomenon. The shear flows around the circular cylinder are sensitive to periodic acoustic excitation with the frequency of transition waves, which generated strong fluctuations and instability in the shear layers around the cylinder, leading to a change in the flow characteristics around the cylinder and the characteristics of vortex induced vibration. Normally this acoustic excitation will be applied to a flow when the vortex induced vibration amplitude became constant. Sound excitation will promote the vortex growth in the early vortex formation behind the cylinder which will cause an increase in frequency of vortex shedding to a value higher than the natural frequency of the mast and that will lead to reduction in the vortex-induced vibration amplitude [11].

1.6 Rationale for this study

As the result of the importance of flow induced vibrations in mast this study is proposed. This study provides a Finite Element tool for design of mast subject to wind. In this study, a tapered mast configuration is considered in order to affect its natural frequencies. The ultimate goal is to create a mast that uses the same amount of material in its construction, but provides higher critical wind speed that guard against potential flow induced vibrations of the mast. In this study a non uniform mass distribution along the length of the mast is considered; three tapered designs with constant thickness are introduced and analyzed for their dynamic response.

CHAPTER II

PROPOSED TAPERED DESIGN AND AN INTRODUCTION INTO SAP2000 AS A FEA TOOL

2.1 Proposed tapered design for a mast subject to wind

One of the most effective ways to improve the dynamic behavior of a mast, with the intension of minimizing its dynamic response to wind, is to ensure that the mast is as stiff and as heavily damped as possible. High stiffness results in high natural frequencies. And since the frequencies of the excitation forces caused by wind are usually low, this discourages any dynamic response, as long as the mast does not have any low natural frequencies close to that of the periodic force caused by the wind. A stiff mast requires high input excitation energy of vibration for a large amplitude response and this requires a long, slow build-up in amplitude under the action of small dynamic wind forces. It is important to note that the random nature of wind and consequently its inability to maintain a high intensity force for long periods of time results in a low level of response in structures with high stiffness.

To increase the stiffness of a mast, a non uniform mass distribution along the longitudinal axis of the mast will be considered in this study. In this thesis mass will be removed from the top of the mast and re-allocated at its base, resulting in a linearly tapered mast. The proposed tapered design maintains the mass, the thickness, and the length of the mast equal to that of the straight design. Increasing the converging angle of the taper of the mast will increase its stiffness, and consequently increasing its natural frequencies. Three tapered designs (I, II, III) will be proposed and analyzed in this thesis. Figure 2.1 shows schematic views of the proposed mast configurations along with a straight mast.

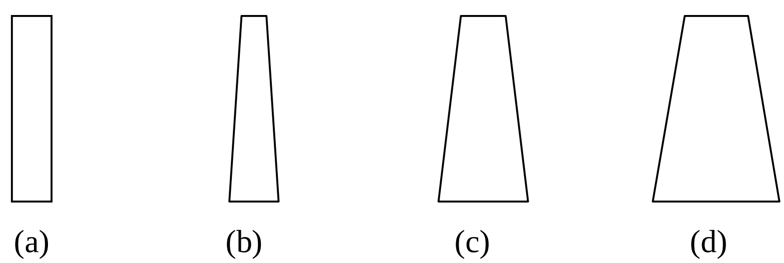

Figure 2.1 Different proposed straight and tapered designs studied in this thesis: (a) Straight design, (b) Tapered design I, (c) Tapered design II, (d) Tapered design III

2.2 Introduction of SAP 2000 as a FEA Tool

To find the response of a vibrating system its equations of motion must be obtained and integrated with respect to time. For a continuous system with infinite degrees of freedom, which is the case for the mast studied in this work, the equations of motion, described as partial differential equations. The solution of such equations in closed form, due to geometric and loading complexity, is rarely obtainable. Therefore other methods and techniques such as Laplace transform methods, matrix methods, and numerical methods are alternative methods to obtain approximate solutions. The challenge in obtaining these approximate solutions is to make sure that the approximation converges to the closed form solutions as the system is modeled with finer and finer discretization in time and spatial parameters of the system. FEA is a numerical method for solving problems of engineering and mathematical physics. The finite element formulation of an engineering problem results in a system of simultaneous algebraic equations for solution, rather than requiring the closed form solution of differential equations [12].

The SAP2000 [13] is a structural analysis program offers the following features:

a. Static and dynamic analysis.
b. Linear and non linear analysis.
c. Dynamic seismic analysis and static pushover analysis.
d. Vehicle live-load analysis for bridges.
e. P-Delta analysis

f. Frame and shell structural elements, including beam column, truss, membrane, and plate behavior.
g. Two- and three- dimensional and axisymmetric solid elements.
h. Nonlinear link and spring elements.
i. Multiple coordinate systems.
j. Many types of constraints.
k. A wide variety of loading options.
l. Alpha-numeric labels.
m. Large capacity.
n. Highly efficient and stable solution algorithms.

These features make SAP 2000 the state of the art in structural analysis programs.

2.3 Capabilities of SAP for structural dynamic analysis

SAP 2000 [13] uses two distinct methods in finding the natural frequencies and mode shapes of a structure. The first method is the Eigenvector modal analysis which involves solving a generalized Eigen value problem to determine the un-damped free vibration mode shapes and natural frequencies of the system. The second method is Ritz-vector modal analysis which is more accurate than the first method because it takes into account the spatial distribution of dynamic loading of structures when finding the mode shapes and its corresponding natural frequencies.

SAP2000 provides Harmonic steady state analysis as a quick tool to find the response of an un-damped structure subjected to periodic loading.

SAP 2000 [13] has also response-spectrum analysis capability, which involves finding the seismic response of a structure subjected to earthquake ground acceleration.

Furthermore, SAP2000 has Time-history analysis capability, which provides the dynamic response of a structure to arbitrary loadings. Damping is taken into account in this analysis. Three types of time-history analysis are available:

a. Linear transient: the structure starts with zero initial conditions or with the conditions at the end of a previous linear transient history that you specify. All elements are assumed to behave linearly for the duration of the analysis.
b. Linear periodic: the initial conditions are adjusted to be equal to those at the end of the period of analysis. All elements are assumed to behave linearly for the duration of analysis.
c. Non linear transient: the structure starts with zero initial conditions or with the conditions at the end of a previous non linear transient history that you specify. The NL Link elements may exhibit non linear behavior during the analysis. All other elements behave linearly. This method is extremely efficient and designed to be used for structural systems which are primarily linear elastic, but which have a limited number of predefined nonlinear elements. In SAP2000, all non linearity is restricted to the NL Link elements.

Yet additionally, SAP2000 provides bridge analysis capability, which can be used to determine the response of a bridge structure due to the weight of vehicle live loads.

2.4 Types of elements used by SAP2000 to analyze a vibrating mast

The mast studied in this work is simply a beam structure. Beam is a long, slender structural member generally subjected to transverse loading that produces significant bending effects.

SAP2000 [13] uses the frame element to model beam-column and truss behavior in planar and three dimensional structures. Frame element uses a general, three dimensional, beam-column formulation which includes the effects of biaxial bending, torsion, axial deformation, and biaxial shear deformation. Frame element is modeled as a straight line connecting two joints, and it normally activates all six degrees of freedom at both its connected joints. Each element has its own local coordinate system for defining section properties and loads. A frame section is a set of material and geometric properties that describe the cross section of one or more frame elements. Sections are defined independently of the frame elements, and are assigned to the elements. Section properties are of two basic types:

a. Prismatic-all properties are constant along the full element length.
b. Non-prismatic- the properties may vary along the element length.

Non-prismatic sections are defined by referring to two or more previously defined prismatic sections.

The isotropic material properties, of a previously defined material, used by the section are:

a. E is the modulus of elasticity
b. G is the shear modulus.
c. γ is the coefficient of thermal expansion.
d. ρ is the mass density
e. ρ_w is the weight density

The geometric properties and section stiffness are:

a. A is the cross sectional area. The axial stiffness of the section is given by ($A\ E$).
b. I_{22}, I_{33} are the second moment of area about the neutral axes of the section 2 and 3 (assuming the first local axis is always directed along the length of the element). The corresponding bending stiffness of the section are given by$(I_{22}\ E)$, $(I_{33}\ E)$.
c. J is the torsional constant, which is the same as polar moment of inertia for circular shapes. The torsional stiffness of the section is given by ($J\ G$)
d. A_{s2}, A_{s3} are the shear areas, for transverse shear in the 1-2 and 1-3 planes, respectively. The corresponding transverse shear stiffnesses of the section are $(A_{s2}\ G)$ and $(A_{s3}\ G)$

Frame element may be prismatic or non-prismatic. The non-prismatic formulation allows the element length to be divided into any number of segments over which properties may vary. The variation of the bending stiffness $(I_{22}E \,\&\, I_{33}E)$ may be linear, parabolic, or cubic over each segment of length. A linear variation in bending stiffness corresponds to linear variation in one of the section dimension. Since the cross-sectional area of the mast under study (circular area) varies linearly along the length of the mast, then both $(I_{22}E)$ and $(I_{33}E)$ will vary linearly. The remaining properties $(A\,E, J\,G, A_{s2}G, A_{s3}G, A\rho_m, A\rho_w)$ will vary linearly between the ends of each segment of a non-prismatic element. Each frame element may be loaded by gravity (in any direction), multiple concentrated loads, multiple distributed loads, loads due to pre-stressing cables, and loads due to temperature change.

Frame element internal forces are the forces and moments that result from integrating the stresses over an element cross section. These internal forces are:

a. The axial force
b. The shear force in 1-2 plane.
c. The shear force in 1-3 plane.
d. The axial torque.
e. The bending moment in the 1-3 plane (about the 2-axis).
f. The bending moment in the 1-2 plane (about the 3-axis)

These internal forces and moments are present at every section along the length of the element as shown in Figure 2.2.

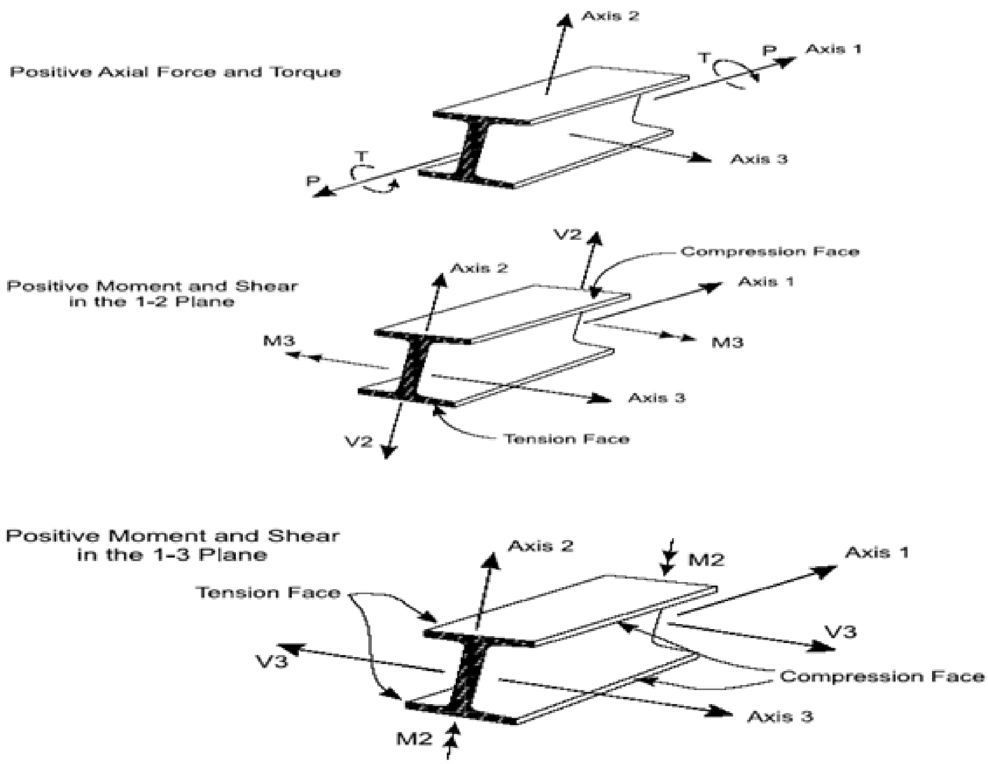

Figure2.2 Frame element integral forces and moments

Finally SAP2000 [13] also calculates the frame element joint forces, which are the concentrated forces and moment acting at the joints of the element. This represents the effect of the rest of the structure upon the frame element, and that cause the translational or rotational deformation of the element along the corresponding degree of freedom.

In the following chapters SAP2000 will be used to build the three proposed tapered design (I, II, III), using a certain number of primary or major non-prismatic elements. Each major non-prismatic element has a finite length; constant thickness and two different outside diameter at each end to represent a varying cross sectional area along the length of the tapered mast. The bottom end of the mast will be fixed (all six degrees of freedom are restrained to zero displacement). One static deflection analysis for both straight and tapered mast will be conducted to validate the results predicted by SAP 2000. Modal analysis will be performed on straight and tapered masts and the 4 first natural frequencies of the configurations will be determined. Finally time history analysis will be conducted in order to determine the dynamic response of tapered and straight masts under wind loading represented by a harmonically varying lift force (vortex shedding excitations).

CHAPTER III

CLOSED FORM STATIC DEFLECTION OF STRAIGHT AND TAPERED MASTS

3.1 Geometric construction of tapered mast configuration

In this study three different tapered mast geometries are developed. Three of the mast dimensions stay unchanged and deflection comparisons are made with that of a straight mast. The dimensions that are held constants for all cases are:

a- The mast volume

b- The mast thickness

c- The mast height

In order to configure the tapered mast under the constraints of constant volume, constant wall thickness, and constant height the outside diameters of the two ends of the mast are varied. Referring to Figure 3.1.a, the volume of the straight mast is:

$$\boldsymbol{V_{st}} = \frac{\boldsymbol{\pi}}{\mathbf{4}}(\boldsymbol{D^2 - d^2}) * \boldsymbol{L} \qquad (\mathbf{3.1})$$

Where, $\boldsymbol{V_{st}}$ is the volume of the straight mast (cm^3)

$\boldsymbol{D}$ is the outside diameter of the straight mast (cm)

$\boldsymbol{d}$ is the inside diameter of the straight mast (cm)

$\boldsymbol{L}$ is the height of the straight mast (cm)

Considering a mast with a height of 2500 cm, the outer and inner diameters of 30 and 28 cm respectively, the total volume of the mast is:

$$\boldsymbol{V_{st}} = \frac{\boldsymbol{\pi}}{\mathbf{4}}[(\mathbf{30^2}) - (\mathbf{28^2})] * \mathbf{2500} = \mathbf{227765\ cm^3} \qquad (\mathbf{3.2})$$

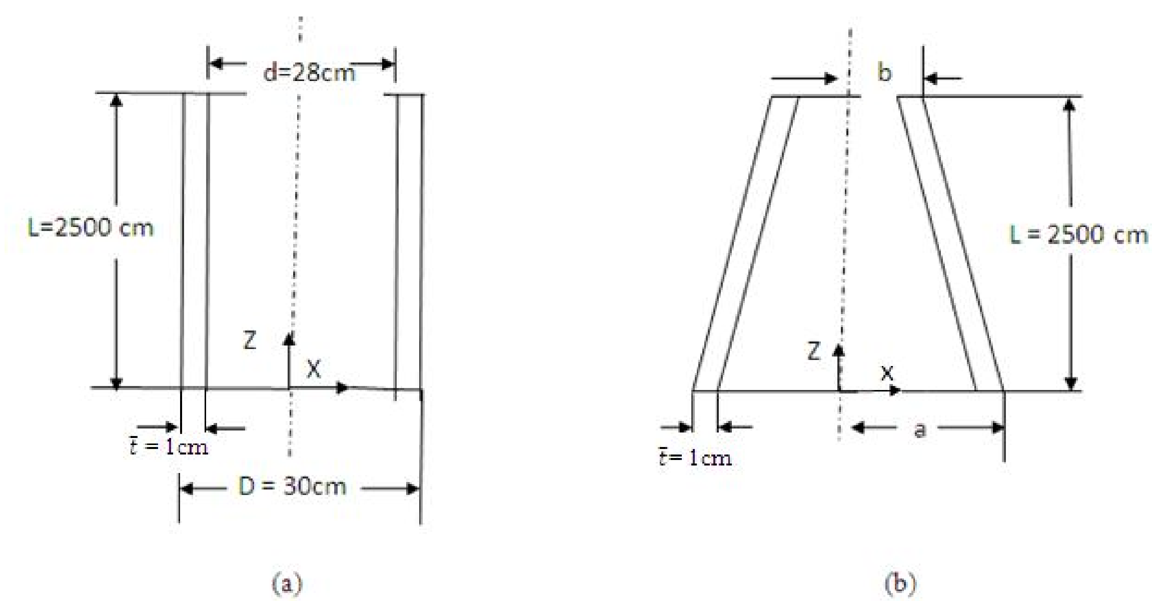

Figure 3.1 Dimensions and coordinate system used for both: (a) Straight mast; (b) Tapered mast

Referring to Figure 3.1.b, the volume of the tapered mast can be calculated as the difference between the volumes of the outer and inner frustums as:

$$\boldsymbol{V_{t1} = \frac{\pi}{3} * L * (a^2 + ab + b^2)} \quad (3.3)$$

$$\boldsymbol{V_{t2} = \frac{\pi}{3} * L * ((a - \bar{t})^2 + (a - \bar{t})(b - \bar{t}) + (b - \bar{t})^2)} \quad (3.4)$$

$$\boldsymbol{V_{tap} = V_{t1} - V_{t2}} \quad (3.5)$$

Where, $\boldsymbol{V_{t1}}$ is the volume of the outer frustum (cm^3)

$\boldsymbol{V_{t2}}$ is the volume of the inner frustum (cm^3)

$\boldsymbol{V_{tap}}$ is the volume of the tapered mast (cm^3)

Using the values of ($\bar{t}$), and (L), and substituting Equation (3.3) and (3.4) into Equation (3.5) yields,

$$\boldsymbol{V_{tap} = V_{t1} - V_{t2} = \pi * 2500 * [a + b - 1]} \quad (3.6)$$

Equate Equation (3.6) for the tapered mast volume to that of the volume of the straight mast described in Equation (3.2), namely $V_{tap} = V_{st}$, yields an expression for the summation of the inner and outer unknown radii of the tapered mast:

$$\boldsymbol{\pi * 2500 * [a + b - 1] = \frac{\pi}{4} * 2500[30^2 - 28^2]} \quad (3.7)$$

$$\boldsymbol{a + b = 30} \quad (3.8)$$

Equation (3.8) is the governing expression for configuring the three tapered mast cases,

Case # I, a = 20 cm, b = 10 cm

Case # II, a = 25 cm, b = 5 cm **(3.9)**

Case # III, a = 28 cm, b = 2 cm

Value of (a) and (b) in Equations (3.9) are used in Table 3.1 that shows the geometric dimensions of the three tapered cases I, II, III, and the straight case, where ID and OD are the inside and outside diameters of the masts respectively.

Case #	Base		Free end	
	OD(cm)	ID(cm)	OD(cm)	ID(cm)
I	40	38	20	18
II	50	48	10	8
III	56	54	4	2
Straight	30	28	30	28

Table 3.1 Geometric Parameters for the tapered and straight masts

3.2 Closed form solutions for static deflections of straight and tapered masts

A powerful simple approach for determining the static deflections of the tapered or straight masts is the use of an energy method called Castigliano's theorem [14]. This theorem states that when forces act upon elastic structural systems, the displacement corresponding to any force, collinear with the force, is equal to the partial derivative of the total strain energy with respect to that force. The terms force and displacement in this statement could be applied equally to moments and angular displacements. Strain or potential energy is the energy stored in an elastic structure as the result of deformation caused by the external forces and moments. If a member is deformed a distance, this energy is equal to the vector dot product of the average force/moment, and the deflection vectors.

The strain energy formulas for a tapered mast Figure 3.1.b under various loading are,

$$\overline{U}_1 = \int_0^L \frac{N^2}{2EA(z)} dz \quad \textit{Tension or compression} \qquad (3.10)$$

$$\overline{U}_2 = \int_0^L \frac{\overline{C}V^2(z)}{2GA(z)} dz \quad \textit{sheer} \qquad (3.11)$$

$$\overline{U}_3 = \int_0^L \frac{\overline{T}^2}{2GJ(z)} dz \quad \textit{Torsion} \qquad (3.12)$$

$$\overline{U}_4 = \int_0^L \frac{M^2(z)}{2EI(z)} dz \quad \textit{Bending moment} \qquad (3.13)$$

Where, $\overline{U}_1, \overline{U}_2, \overline{U}_3,$and $\overline{U}_4$ are the strain energies due to tension/compression, shear, torsion and bending moment respectively. The parameters of Equations (3.10) through (3.13) are defined as:

N is the axial force, causing tension or compression of the mast(N).

$V(z)$ is the sheer force, varying alone the mast's length (N).

$\overline{T}$ is the torque, putting the mast under torsion (N.cm).

$M(z)$ is the bending moment, varying in the z-direction along the mast's longitudinal axis (N.cm).

$A(z)$ is the cross sectional area of the tapered or straight mast, varying in the z-direction along the mast's longitudinal axis (cm^2).

$J(z)$ is the Polar second moment of the mast's cross sectional area, varying in the z-direction along the mast's longitudinal axis (cm^4).

$\boldsymbol{I(z)}$ is the Rectangular second moment of the mast's a cross sectional area about the neutral axis, varying in the z-direction along the mast's longitudinal axis (cm^4).

$\boldsymbol{L}$ is the length of the tapered or straight mast (cm).

$\boldsymbol{E}$ is the Modules of elasticity. Aluminum is the material used (N/cm^2)

$\boldsymbol{G}$ is the Modulus of rigidity, aluminum is the material used (N/cm^2)

$\boldsymbol{\bar{C}}$ is a dimensionless correction factor whose value depends upon the shape of the mast's cross section, for tubular cross section ($\bar{C}$ = 2).

In the most general case the total strain energy $\bar{U}_t$ of a tapered or straight mast is the summation of all strain energy various loadings contributions. Mathematically,

$$\boldsymbol{\bar{U}t = \int_0^L \left(\frac{N^2}{2EA(z)} + \frac{\bar{C}V^2(z)}{2GA(z)} + \frac{\bar{T}^2}{2GJ(z)} + \frac{M^2(z)}{2EI(z)} \right) dz} \qquad \textbf{(3.14)}$$

For a tapered or straight mast loaded with a concentrated load *(F)* at its tip, and acting in the positive x-direction, shown in Figure 3.2, the total strain energy equation (Eqn.3.14) will reduce to:

$$\boldsymbol{\bar{U}t = \int_0^L \frac{\bar{C}V^2(z)}{2GA(z)} + \frac{M^2(z)}{2EI(z)} dz} \qquad \textbf{(3.15)}$$

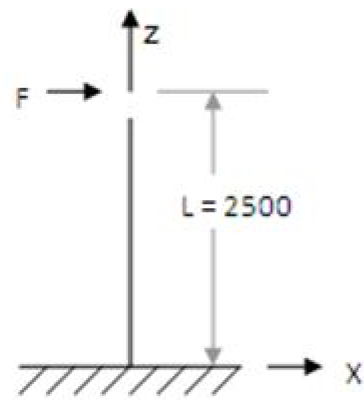

Figure 3.2 Tapered or straight mast with concentrated load *(F)* acting at the tip or free end of the mast in the positive x-direction.

Under this concentrated load (*F*) condition

$$\boldsymbol{V(z) = F} \qquad \mathbf{(3.16.}\boldsymbol{a}\mathbf{)}$$

$$\boldsymbol{M(z) = F(z - L)} \qquad \mathbf{(3.16.}\boldsymbol{b}\mathbf{)}$$

$$\boldsymbol{\bar{C} = 2\ (For\ tubular\ cross\ section)} \qquad \mathbf{(3.16.}\boldsymbol{c}\mathbf{)}$$

$$\boldsymbol{L = 2500\ (cm)} \qquad \mathbf{(3.16.}\boldsymbol{d}\mathbf{)}$$

Substituting values from Equation (3.16) back into Equation (3.15) yields:

$$\boldsymbol{\bar{U}t = \int_{0}^{2500} \left(\frac{F^2}{GA(z)} + \frac{F^2 (z - 2500)^2}{2EI(z)} \right) dz} \qquad \mathbf{(3.17)}$$

Castigliano's theorem will be applied now to find the static deflection of the tip of the tapered or straight mast in the direction of the force (*F*), the x-direction. The value of the deflection is equal to the partial derivative of the total strain energy with respect to the force (*F*) mathematically.

$$\boldsymbol{\delta} = \frac{\partial \bar{U}_t}{\partial F} = \int_0^{2500} \left(\frac{2F}{GA(z)} + \frac{F(z-2500)^2}{EI(z)}\right) dz \qquad (3.18)$$

Where, $\boldsymbol{\delta}$ is the static deflection of the tip of the tapered or straight mast in the direction of the force *F*, which is the x-direction. Equation (3.18) is the general equation for the deflection of the tip of the mast, and it is important to note that δ is a function of F.

The tapered beam studied in this thesis has a varying tabular cross section, therefore the mast cross-section, $A(z)$, and the area-moment-of-inertia, $I(z)$, are both functions of the variable z, and could be expressed according to Equations (3.19.a) and (3.19.b) as:

$$A(z) = \pi\,[r_o^2(z) - r_i^2(z)] \qquad (3.19.a)$$

$$I(z) = \frac{\pi}{4}\,[r_o^4(z) - r_i^4(z)] \qquad (3.19.b)$$

Where, $\boldsymbol{r_o(z)}$ is the outside radius of the tubular cross sectional area of the tapered mast, taken at any point, in the z -direction; along its longitudinal axis.

$r_i\,(z)$ is the inside radius of the tabular cross sectional area of the tapered mast, taken at any point, in the z -direction, along its longitudinal axis.

$$For\ case\#I, r_o(z) = \frac{5000-z}{250}, r_i(z) = \frac{4750-z}{250} \qquad (3.20.a)$$

$$For\ case\#II, r_o(z) = \frac{3125-z}{125}, r_i(z) = \frac{3000-z}{125} \qquad (3.20.b)$$

$$For\ case\#III, r_o(z) = \frac{2692-z}{96}, r_i(z) = \frac{2596-z}{96} \qquad (3.20.c)$$

$$\boldsymbol{For\ straight\ mast\ case, r_o(z) = \frac{D}{2} = 15, r_i(z) = \frac{d}{2} = 14} \qquad (3.20.d)$$

Table 3.2 provides cross sectional properties required to evaluate the static deflection of the tapered or straight mast, Equation (3.18), and in this study Aluminum is the material used for the mast.

Case#	$A(z)$ (cm^2)	$I(z)$ (cm^4)
I	$\pi\left[\frac{(5000-z)^2}{250} - \frac{(4750-z)^2}{250}\right]$	$\frac{\pi}{4}\left[\frac{(5000-z)^4}{250} - \frac{(4700-z)^4}{250}\right]$
II	$\pi\left[\frac{(3125-z)^2}{125} - \frac{(3000-z)^2}{125}\right]$	$\frac{\pi}{4}\left[\frac{(3125-z)^4}{125} - \frac{(3000-z)^4}{125}\right]$
III	$\pi\left[\frac{(2692.3-z)^2}{96.15} - \frac{(2596.15-z)^2}{96.15}\right]$	$\frac{\pi}{4}\left[\frac{(2692.3-z)^4}{96.15} - \frac{(2596.15-z)^4}{96.15}\right]$
Straight	364.24	1.53345E5

Table 3.2 Cross sectional properties $A(z)$**,** and $I(z)$ for tapered and straight designs.

To avoid the algebraic tedium in solving Equation (3.18) Maple 8 software will be used to find the static deflection. The parameters of Equation (3.18) for the three tapered and straight cases, the command lines, and the solution are in appendix (A).

The material properties, *G* and *E* for an assumed aluminum mast are:

$$\boldsymbol{G_{AL} = 2.8904E6\ (\mathrm{N/cm^2})} \qquad \textbf{(3.21.a)}$$

$$\boldsymbol{E_{AL} = 7.515E6\ (\mathrm{N/cm^2})} \qquad \textbf{(3.21.b)}$$

Substituting the values of G and E from Equations (3.21.a) and (3.21.b) in the expressions for the static deflections of the mast according to the Castigliano's theorem, Equation (3.18), yields the following numerical values for the 4 cases:

$$\begin{aligned} &\boldsymbol{Case\ \#I},\ \boldsymbol{\delta_1} = \mathbf{0.04947984992} * \boldsymbol{F} \\ &\boldsymbol{Case\ \#II},\ \boldsymbol{\delta_2} = \mathbf{0.004504452184} * \boldsymbol{F} \\ &\boldsymbol{Case\ \#III},\ \boldsymbol{\delta_3} = \mathbf{0.05679230030} * \boldsymbol{F} \\ &\boldsymbol{Straight\ case},\ \boldsymbol{\delta_{str}} = \mathbf{0.07229593626} * \boldsymbol{F} \end{aligned} \tag{3.22}$$

3.3 Result of static deflections for straight and tapered masts

Equation (3.22) is used to calculate the static deflection of the tip of the straight or tapered masts for different values of force F, as shown in Table 3.3

Case#	F = 250 (N)	F = 400 (N)	F = 700 (N)
	δ(cm)	δ(cm)	δ(cm)
I	12.3699	19.7919	34.635
II	11.2611	18.017	31.5311
III	14.198	22.7169	39.754
Straight	18.074	28.9183	50.607

Table 3.3 The static deflection of tapered and straight cases, using closed form formula under different load *(F)* values

It is obvious from Table 3.3 that increasing the load increased the static deflection for all four designs. This is due to the fact that increasing the load value will increase the bending moment, and consequently increasing the static deflection.

CHAPTER IV

STATIC DEFLECTION ANALYSIS OF STRAIGHT AND TAPERED MASTS USING SAP2000

4.1 Use of SAP2000 for static deflection analysis of a straight mast

In order to become familiar with SAP, static deflection analyses of straight and tapered beams are carried out in this chapter. To use SAP2000, and obtain the static deflection of a straight mast, three major steps required:

a- Building the model geometry, and its restraints.

b- Creating and setting the parameters for static analysis.

c- Reviewing the results (deflection) and compare them with the results of known closed-form solutions.

To build the model of the straight the mast, the following steps are to be taken:

A1 Define the system of units to be used (N, m, c) throughout the analysis. Also define the grid system, which is a set of intersecting lines used to aid in drawing of the model, for the global coordinate system, as shown in Figure 4.1.

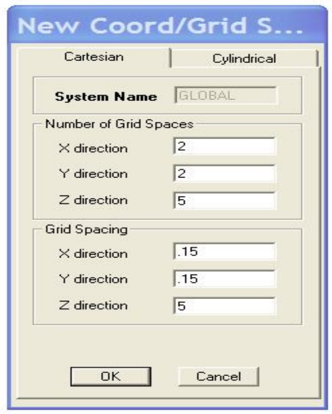

Figure 4.1 Define the Grid System of the Global Coordinate System.

A2 Define material and section properties of frame elements that are to be used to build the model. An Aluminum alloy material will be used in all analyses throughout the thesis with isotropic properties as shown in Figure 4.2. Also one prismatic frame section (PIPE) is defined for the straight mast as shown in Figure 4.3.

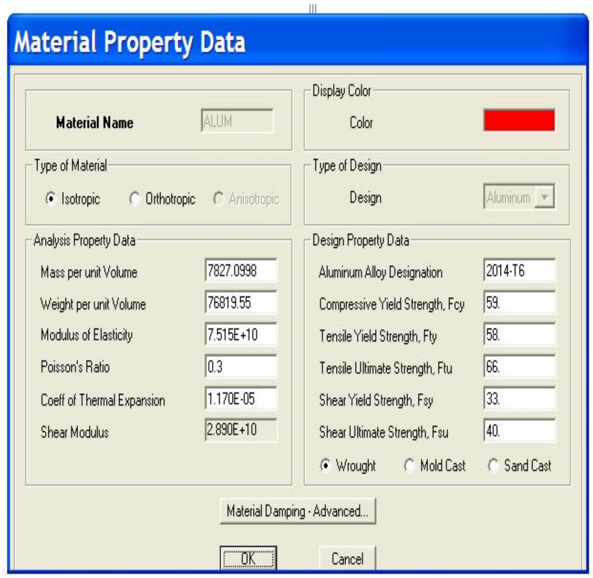

Figure 4.2 Isotropic material properties for the Aluminum Alloy used in all analyses throughout the thesis.

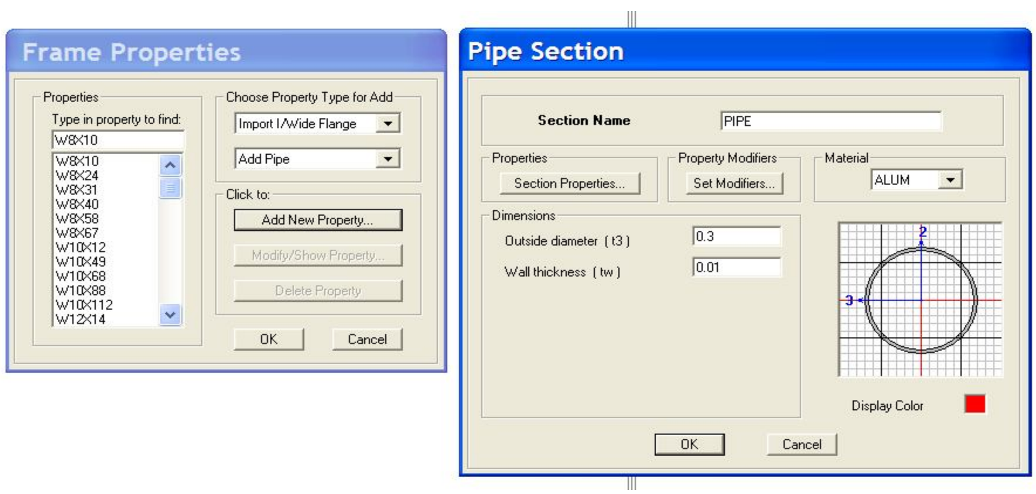

Figure 4.3 Defining the prismatic frame section (PIPE) properties of frame element used in building the straight mast. The outside diameter is 0.3 m and the constant wall thickness is 0.01m .

Once the outside diameter (0.3 m) and thickness (0.01m) of the section (PIPE) are chosen, SAP2000 automatically calculates six major geometric properties (A, J,I_{22}, I_{33}, A_{s2}, A_{s3}), which were previously defined in Chapter 2, as shown in Figure 4.4.

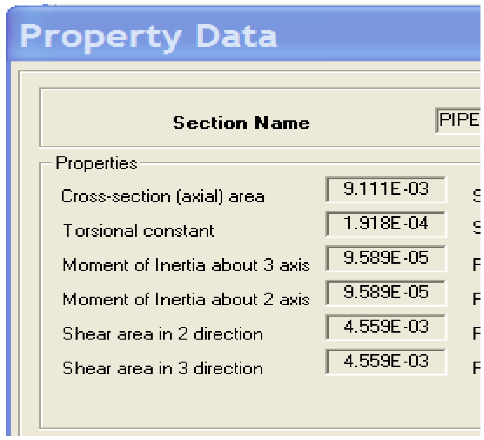

Figure 4.4 Six major geometric properties for the (PIPE) Section automatically calculated by SAP2000

A3 Draw the model as one prismatic frame element of 25 meters in length using frame section (PIPE) as shown in Figure 4.5. Normally, the three translational and three rotational degrees of freedom at each end of the frame element are continuous with those of the joint, and hence with those of all other elements connected to that joint. However it is possible to release (disconnect) one or more of the element's degrees of freedom from the joint when it is known that the corresponding element force or moment is zero (force or moment release).

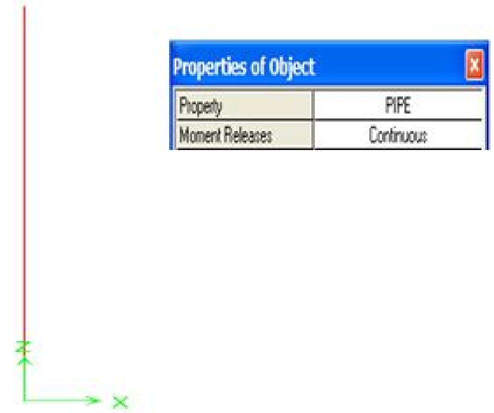

Figure 4.5 Drawing a one frame element of 25m in length, with no releases.

A4 Define the fixed support at the bottom of the mast. If the displacement of a joint along any one of its degrees of freedom is known, such as at a fixed support point, that degree of freedom is restrained. For the previously drawn mast of one prismatic element, its bottom joint (0, 0, 0) displacement will be restrained in the three translational and three rotational degrees of freedom to a value of zero. This kind of restrain represents a fixed support as shown in Figure 4.6.

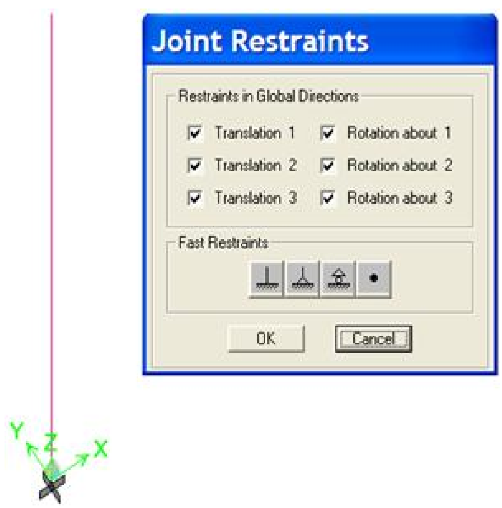

Figure 4.6 Defining the fixed support at the bottom of the mast (X=Y=Z= 0) by restraining all six degrees of freedom at the joint.

To perform the static analysis, the following steps need to be carried out:

B1 Define the static analysis load case as shown in Figure 4.7. In the example presented here the static load is a spatial distribution of loads upon the structure. Each load case has a specific unique name (St. Mast). Specify the design type for the load case (DEAD, LIVE, SNOW, THERMAL LOAD, and so on) used by the program to create design combinations. In our case the design type is (DEAD). Self weight multiplier is a scale factor that multiplies the weight of every element in the structure and applies it as a force in the gravity direction (negative global z-direction). So to include the weight of the mast in load case (St. Mast) a value of one is set for the self weight multiplier. Finally auto lateral load option stays empty because it is only used with load cases of design-type WIND or QUAKE.

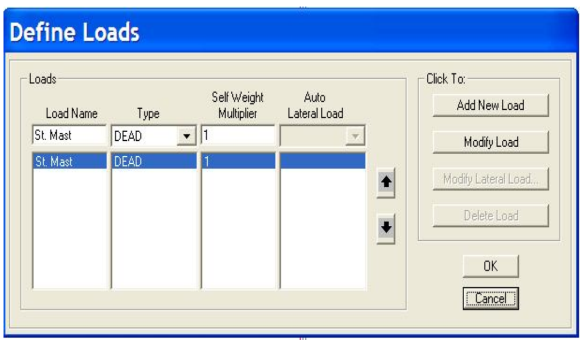

Figure 4.7 Defining static analysis load case (St. Mast), which is DEAD type load with a self weight multiplier value of one.

B2 Specify the spatial distribution of load case (St. Mast) used in the static analysis. A concentrated load applied at the free end of the mast in the positive x-direction will be used for the static analysis as shown in Figure 4.8.

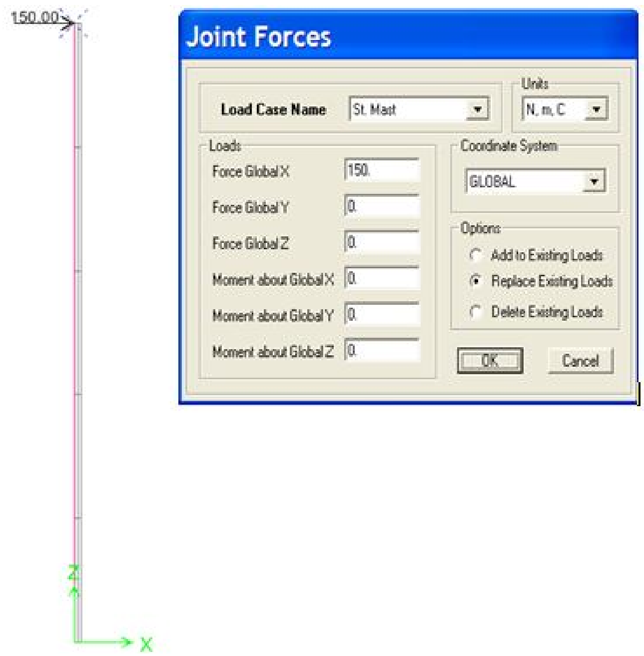

Figure 4.8 Defining the spatial distribution of load case (St. Mast), which is a concentrated load applied at the free end of the mast in the positive global X- direction(Ex: 150 N in the positive X- Dir.)

B3 Divide the single frame element previously defined into 4000 equal length frame elements as shown in Figure 4.9. Use a value of one for the ratio of the last element length to the first element length (equal frame elements length).

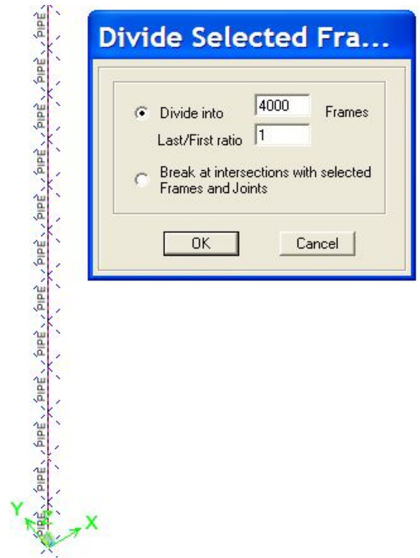

Figure 4.9 Dividing the one frame element into 4000 elements of the same length and same frame section (PIPE)

B4 Setting the static analysis case (St. Mast Static) parameters; linear static analysis type is used where material or geometric non linear behavior are ignored. In solving for the response (deflection of the mast) the stiffness of the unstressed structure will be used as shown in Figure 4.10. Under the applied loads, two load options exist, the load case option, marked as: "load" and the built-in acceleration load option, marked as: "accel". In order to continue with the static force analysis, we should select the load type, namely "load" with the load case (St. Mast), and choose a scale factor of value of one. The "Scale Factor" is a load multiplier used before adding it to other loads applied under that static load case.

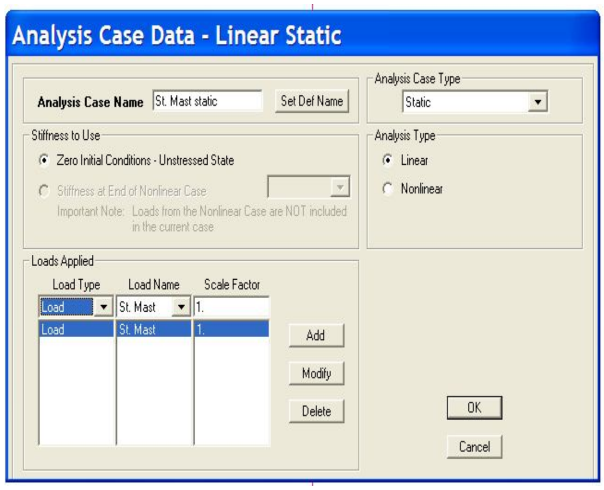

Figure 4.10 Setting the parameters of linear static analysis case (St. Mast Static)

Once all previous steps are completed, run the static analysis case (St. Mast Static) as shown in Figure 4.11. Here in this study the total number of frame elements is set to be 4000 with 24000 active degrees of freedom that involves solving a system of 24000 equilibrium linear equations represented by

$$\boldsymbol{K}.\boldsymbol{u} = \boldsymbol{r} \qquad (4.1)$$

Where,

$\boldsymbol{K}$ is the structural stiffness matrix.

$\boldsymbol{r}$ is the vector of the applied loads.

$\boldsymbol{u}$ is the vector of the resulting displacements.

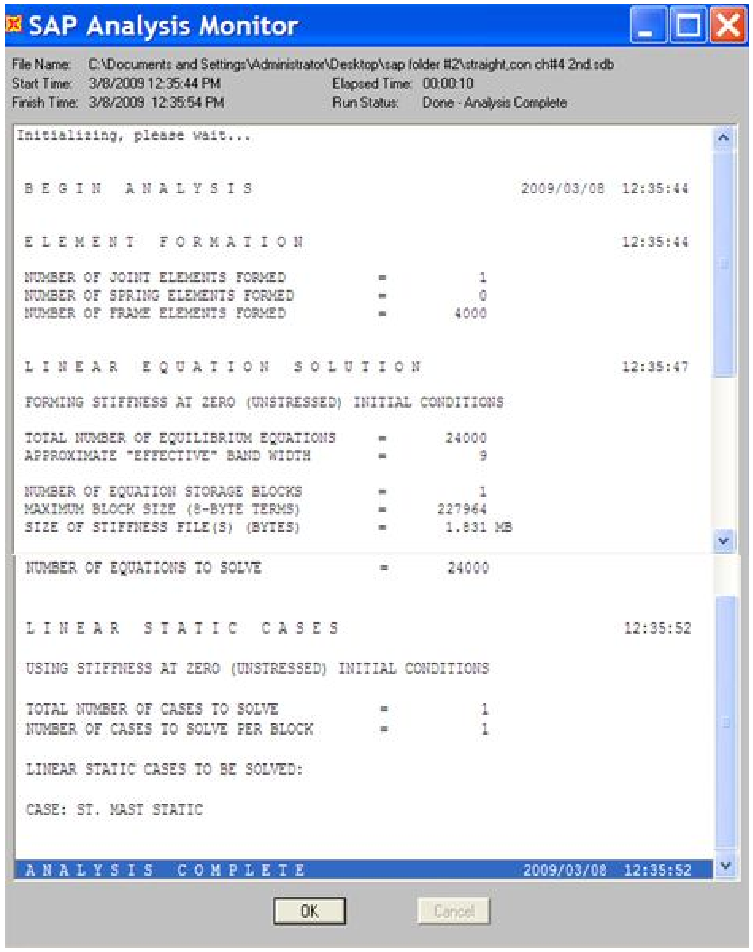

Figure 4.11 Running static analysis case (St. Mast Static).

Finally to review the displacement of the mast at any joint or point, simply select that point and the response along all six degrees of freedom for that point is calculated as shown in Figure 4.12.

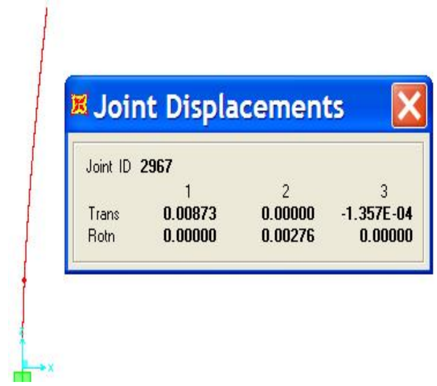

Figure 4.12 The response along all six degrees of freedom at any point along the mast.

4.2 Use of SAP2000 for static deflection analysis of a tapered mast

The same steps explained and carried out in previous section are applicable for the static analysis of a tapered mast, but with a few changes taking place.

When defining grid system, the number of grid spaces in the z-direction is chosen to be 20, with grid spacing of 1.25 m to give a total of 25 m in the z-direction. This will allow drawing of the primary (20) non-prismatic frame elements for this case. Each non-prismatic element is 1.25 m in length for modeling of the whole length of any of the three tapered mast cases (I, II, III) defined in chapter (3). For any of the tapered mast cases (I, II, III) modeled, each of the (20) non-prismatic elements is chosen to be 1.25 m in length with two different diameters at the ends of the element. The outside diameter of the primary non-prismatic element will be varying (decreasing) linearly along its length. The bending stiffness $(I_{22}E)$ and $(I_{33}E)$ defined in section (2.4), vary linearly over the non-prismatic element length in correspondence to a linear variation in section outside diameter along the element length. Two prismatic sections are defined for each primary non-prismatic element as shown in Figure 4.13. All the (21) prismatic sections defined have the same wall thickness of (1 cm) as shown in Figure 4.14.

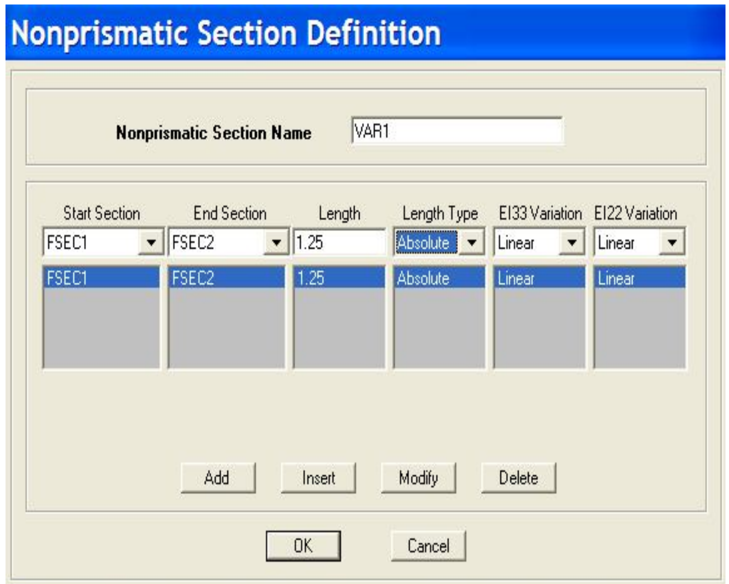

Figure 4.13 The non-prismatic frame element (VAR1) requires the definition of two prismatic sections(FSEC1 & FSEC2). It has an absolute length of 1.25 m and the bending stiffness (EI22 &EI33) vary linearly over the element length.

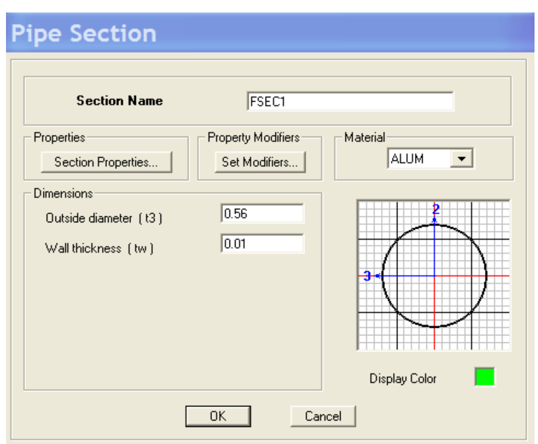

Figure 4.14 In all the (21) prismatic sections used in defining the (20) non-Prismatic elements, the wall thickness stays (0.01 m).

Each of the primary non-prismatic elements is divided into 200 elements to have a total of 4000 non-prismatic elements (the same number of elements used in the straight mast analysis. The methodology that was used in deciding the number of primary non-prismatic elements was based on building four models with different number of non-prismatic elements (1,5,10, and 20), and in each of the four cases the primary elements were divided into a total of (4000) non-prismatic element. Moving from one element to 20 elements the value of response converges and the percentage of change in response beyond twenty elements is small and negligible. To illustrate that, (34) elements in case (I) and (60) elements in case (III) were used to build the model. No major changes in the value of response going from (20) elements to (34) or (60) elements as shown in table 4.1. As a result, using twenty primary non-prismatic elements in modeling the tapered mast leads to a very accurate static analysis for mast deflection.

	Static deflection at the tip of tapered mast δ(cm) using SAP2000					
No. of non-prismatic elements	1	5	10	20	34	60
I	18.106	12.588	12.423	12.383	12.374	———
II	19.134	11.949	11.431	11.302	———	———
III	22.859	16.942	15.224	14.484	———	14.227

Table 4.1 The static deflection of the three tapered cases (I, II, III) using SAP2000 the value of concentrated load is (250N) acting at the tip of the mast. The total number of non-prismatic elements after division is 4000 elements.

4.3 Parametric study of straight and tapered masts using SAP2000

The straight and tapered (I, II, III) designs introduced in chapter (3) are utilized in this parametric study. A concentrated load of different values (250, 400, and 700 N) acting at the tip of the mast will be analyzed for static deflection using SAP2000. A total of 4000 frame elements were used for meshing. Table 4.2 shows in static deflection (δ) of all four designs for three different loadings.

Case #	F = 250(N)	F = 400(N)	F = 700 (N)
	δ(cm)	δ(cm)	δ(cm)
I	12.383	19.813	34.672
II	11.302	18.083	31.646
III	14.484	23.173	40.552
Straight	18.073	28.917	50.605

Table 4.2 The static deflection (δ) of tapered (I, II, III) and straight mast cases Under different loading values using SAP2000

4.4 Validation of FEA of SAP2000 using the closed form solution

The set of equations derived in chapter three (Eqn. 3.22), using Castigliano's theorem for static deflection, for all four designs (straight, I, II, III) will be used to validate the FEA using SAP2000 as shown in Table 4.3

Case #	F = 250(N)		F = 400(N)		F = 700(N)	
	δ(cm)		δ(cm)		δ(cm)	
	SAP	(Eqn.3.22)	SAP	(Eqn.3.22)	SAP	(Eqn.3.22)
I	12.383	12.3699	19.813	19.7919	34.672	34.635
II	11.302	11.2611	18.083	18.017	31.646	31.5311
III	14.484	14.198	23.173	22.7169	40.552	39.754
Straight	18.073	18.074	28.917	28.9183	50.605	50.607

Table 4.3 The validation of SAP2000 using the closed form solution for static deflection (δ)

It is obvious from comparing the two sets of results (SAP2000, and closed form solution) that the difference in value is very small and negligible. This leads to the conclusion that SAP2000 will give very accurate results for static deflection using twenty primary non-prismatic elements (with a total of 4000 divided frame elements).

CHAPTER V

THE NATURAL FREQUENCIES OF A STRAIGHT MAST

5.1 Natural frequencies of a straight mast by closed form solutions

The equations governing the natural frequency of a straight mast is derived in this chapter [15]. Figure 5.1.a shows the free body diagram of the mast in bending. Figure 51.b shows an element of the mast, where $M(z,t)$ is the bending moment, $V(z,t)$ is the shear force, and $\bar{f}(z,t)$ is the external force per unit length of the mast.

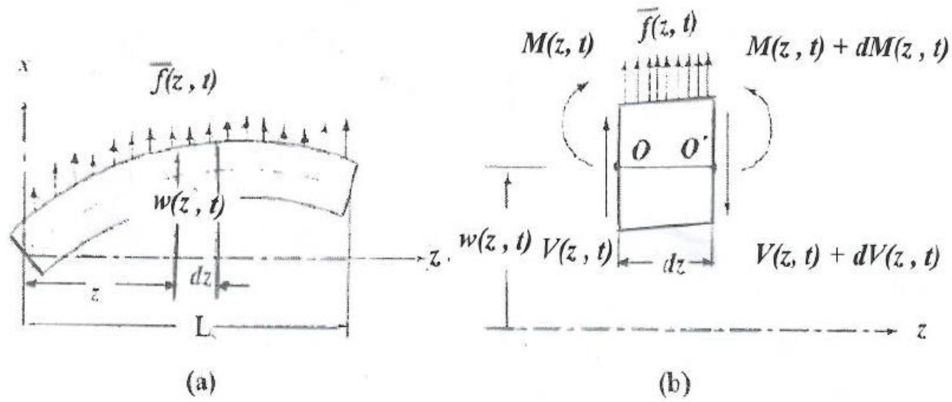

Figure 5.1 (a) A straight mast in bending, (b) A free body diagram of an element of the mast.

The force equation of motion in the z-direction yields:

$$-(V + dV) + \bar{f}(z, t)dz + V = \rho A \ dz \ \frac{\partial^2 w}{\partial t^2} \ (z, t) \qquad (5.1)$$

Where, $\boldsymbol{\rho}$ is the mass density and $\boldsymbol{A}$ is the cross-sectional area of the straight mast. The moment equation of motion about the y-axis passing through point "O" in Figure 5.1.b yields:

$$(M + dM) - (V + dV)dz + \bar{f}(z, t)dz \ \frac{dz}{2} - \ M = 0 \qquad (5.2)$$

By noting that

$$dV = \frac{\partial V}{\partial z} \ dz \quad and \quad dM = \frac{\partial M}{\partial z} \ dz$$

And disregarding terms involving second powers in dz, Equations (5.1) and (5.2) can be written as:

$$\frac{-\partial V}{\partial z} \ (z, t) + \bar{f}(z, t) = \rho A \ \frac{\partial^2 w}{\partial t^2} \ (z, t) \qquad (5.3)$$

$$\frac{\partial \boldsymbol{M}}{\partial \boldsymbol{z}}\,(\boldsymbol{z},\boldsymbol{t}) - \boldsymbol{V}(\boldsymbol{z},\boldsymbol{t}) = \mathbf{0} \qquad (\mathbf{5.4})$$

Equation (5.4) can be rewritten as

$$\boldsymbol{V}(\boldsymbol{z},\boldsymbol{t}) = \frac{\partial \boldsymbol{M}}{\partial \boldsymbol{z}}\,(\boldsymbol{z},\boldsymbol{t}) \qquad (\mathbf{5.5})$$

Substituting Equation (5.5) into Equation (5.3) gives

$$\frac{-\partial^2 \boldsymbol{M}}{\partial \boldsymbol{z}^2}\,(\boldsymbol{z},\boldsymbol{t}) + \bar{\boldsymbol{f}}(\boldsymbol{z},\boldsymbol{t}) = \boldsymbol{\rho A}\,\frac{\partial^2 \boldsymbol{w}}{\partial \boldsymbol{t}^2}\,(\boldsymbol{z},\boldsymbol{t}) \qquad (\mathbf{5.6})$$

From the elementary theory of bending of beams,

$$\boldsymbol{M}(\boldsymbol{z},\boldsymbol{t}) = \boldsymbol{E}\,\boldsymbol{I}(\boldsymbol{z})\frac{\partial^2 \boldsymbol{w}}{\partial \boldsymbol{z}^2}\,(\boldsymbol{z},\boldsymbol{t}) \qquad (\mathbf{5.7})$$

Where, E is Young's modulus and $I(z)$ is the moment of inertia of beam cross section about the y-axis. Substituting Equation (5.7) into Equation (5.6) gives

$$\frac{\partial^2}{\partial \boldsymbol{z}^2}\left[\boldsymbol{E}\,\boldsymbol{I}(\boldsymbol{z})\,\frac{\partial^2 \boldsymbol{w}}{\partial \boldsymbol{z}^2}(\boldsymbol{z},\boldsymbol{t})\right] + \boldsymbol{\rho A}\,\frac{\partial^2 \boldsymbol{w}}{\partial \boldsymbol{t}^2}\,(\boldsymbol{z},\boldsymbol{t}) = \bar{\boldsymbol{f}}(\boldsymbol{z},\boldsymbol{t}) \qquad (\mathbf{5.8})$$

Now for free vibration, $\bar{f}(z,t) = 0$, and for a straight mast where $I(z)$ is constant Equation (5.8) becomes

$$\boldsymbol{E}\,\boldsymbol{I}\,\frac{\partial^4 \boldsymbol{w}}{\partial \boldsymbol{z}^4}(\boldsymbol{z},\boldsymbol{t}) + \boldsymbol{\rho A}\,\frac{\partial^2 \boldsymbol{w}}{\partial \boldsymbol{t}^2}\,(\boldsymbol{z},\boldsymbol{t}) = \mathbf{0} \qquad (\mathbf{5.9})$$

Rearranging terms Equation (5.9) becomes

$$\boldsymbol{\alpha}^2\,\frac{\partial^4 \boldsymbol{w}}{\partial \boldsymbol{z}^4}(\boldsymbol{z},\boldsymbol{t}) + \frac{\partial^2 \boldsymbol{w}}{\partial \boldsymbol{t}^2}\,(\boldsymbol{z},\boldsymbol{t}) = \mathbf{0} \qquad (\mathbf{5.10})$$

Where,

$$\boldsymbol{\alpha} = \sqrt{\frac{\boldsymbol{EI}}{\boldsymbol{\rho A}}} \qquad (\mathbf{5.11})$$

Equation (5.10) is the equation of motion for the un-damped free lateral vibration of a straight mast. Four boundary conditions and two initial conditions are needed for finding a unique solution for $w(z,t)$. The solution of Equation (5.10) can be found using the method of separation of variables as

$$\boldsymbol{w(z,t) = W(z)T(t)} \qquad \textbf{(5.12)}$$

Substituting Equation (5.12) into Equation (5.10) and rearranging terms gives

$$\frac{\boldsymbol{\alpha^2}}{\boldsymbol{W(z)}} \frac{\boldsymbol{d^4W(z)}}{\boldsymbol{dz^4}} = -\frac{\mathbf{1}}{\boldsymbol{T(t)}} \frac{\boldsymbol{d^2T}}{\boldsymbol{dt^2}} = \boldsymbol{h} = \boldsymbol{\omega^2} \qquad \textbf{(5.13)}$$

Where, h is a positive constant equals square the natural frequency of vibration ω^2. Equation (5.13) can be rewritten as two equations

$$\frac{\boldsymbol{d^4W(z)}}{\boldsymbol{dz^4}} - \boldsymbol{\beta^4 W(z) = 0} \qquad \textbf{(5.14.}\boldsymbol{a}\textbf{)}$$

$$\frac{\boldsymbol{d^2T(t)}}{\boldsymbol{dt^2}} + \boldsymbol{\omega^2\, T(t) = 0} \qquad \textbf{(5.14.}\boldsymbol{b}\textbf{)}$$

Where,

$$\boldsymbol{\beta^4} = \frac{\boldsymbol{\omega^2}}{\boldsymbol{\alpha^2}} = \frac{\boldsymbol{\rho A \omega^2}}{\boldsymbol{EI}} \qquad \textbf{(5.15)}$$

Our main concern is with Equation (5.14.a), which will be used to find the natural frequencies of the straight mast. The solution of Equation (5.14.a) can be written as

$$\boldsymbol{W(z) = C\, e^{sz}} \qquad \textbf{(5.16)}$$

Where, $\boldsymbol{C}$ and $\boldsymbol{s}$ are constants to be determined. Substituting Equation (5.16) into Equation (5.14.a) gives the auxiliary equation as:

$$\boldsymbol{s^4 - \beta^4 = 0} \qquad \textbf{(5.17)}$$

The roots of this equation are

$$\boldsymbol{s_{1,2} = \pm\beta} \quad , \quad \boldsymbol{s_{3,4} = \pm i\beta} \qquad \textbf{(5.18)}$$

The solution of Equation (5.14.a) becomes

$$W(z) = C_1 e^{\beta z} + C_2 e^{-\beta z} + C_3 e^{i\beta z} + C_4 e^{-i\beta z} \qquad (5.19)$$

Where, C_1, C_2, C_3, and C_4 are constants. Equation (5.19) can also be written as

$$W(z) = C_1 \cos \beta z + C_2 \sin \beta z + C_3 \cosh \beta z + C_4 \sinh \beta z \qquad (5.20)$$

The constants C_1 to C_4 and the value of β can be determined by imposing the appropriate boundary conditions. The natural frequencies of the straight mast are computed from Equation (5.15) as

$$\omega = \beta^2 \sqrt{\frac{EI}{\rho A}} = (\beta L)^2 \sqrt{\frac{EI}{\rho A L^4}} \qquad (5.21)$$

Where, L is the length of the mast.

For a cylindrical shell straight mast, the cross sectional area A , and the moment of inertia of the beam cross section about the y-axis I are

$$A = \frac{\pi}{4}(D^2 - d^2) \qquad (5.22)$$

$$I = \frac{\pi (D^2 - d^2)}{64} \qquad (5.23)$$

Where, D is the outside diameter of the straight mast.

d is the inside diameter of the straight mast.

For typical mast, there will be an infinite number of normal modes and one natural frequency associated with each normal mode. For fixed end at $(z = 0)$ and free end at $(z = L)$, the boundary conditions are

$$W(0) = 0 \qquad (5.24.a)$$

$$\frac{dW}{dz}(0) = 0 \qquad (5.24.b)$$

$$EI\,\frac{d^2W}{dz^2}(L) = 0 \;\; or \;\; \frac{d^2W}{dz^2}(L) = 0 \qquad (5.24.c)$$

$$\frac{d}{dz}\left(EI\,\frac{d^2W}{dz^2}(L)\right) = 0 \;\; or \;\; \frac{d^3W}{dz^3}(L) = 0 \qquad (5.24.d)$$

Substituting condition (5.24.a) into Equation (5.20) gives

$$C_1 + C_3 = 0 \qquad (A)$$

The derivative of Equation (5.20) with respect to (z) gives

$$\frac{dW}{dz} = \beta\,(-C_1 \sin\beta z + C_2 \cos\beta z + C_3 \sinh\beta z + C_4 \cosh\beta z) \qquad (B)$$

Substituting condition (5.24.b) into (B) gives

$$\beta\,(C_2 + C_4) = 0 \qquad (C)$$

Thus $W(z)$ becomes

$$W(z) = C_1\,(\cos\beta z - \cosh\beta z) + C_2(\sin\beta z - \sinh\beta z) \qquad (D)$$

The first, second, and third derivatives of $W(z)$ with respect to (z) are

$$\frac{dW}{dz} = C_1\beta\,(-\sin\beta z - \sinh\beta z) + C_2\beta(\cos\beta z - \cosh\beta z) \qquad (E)$$

$$\frac{d^2W}{dz^2} = C_1\beta^2\,(-\cos\beta z - \cosh\beta z) + C_2\beta^2(-\sin\beta z - \sinh\beta z) \qquad (F)$$

$$\frac{d^3W}{dz^3} = C_1\beta^3\,(\sin\beta z - \sinh\beta z) + C_2\beta^3(-\cos\beta z - \cosh\beta z) \qquad (G)$$

Substituting condition (5.24.c) into (F) gives

$$\frac{d^2W}{dz^2}(L) = C_1\beta^2(-\cos\beta L - \cosh\beta L) + C_2\beta^2(-\sin\beta L - \sinh\beta L) = 0 \qquad (H)$$

Substituting condition (5.24.d) into (G) gives

$$\frac{d^3W}{dz^3}(L) = C_1\beta^3(\sin\beta L - \sinh\beta L) + C_2\beta^3(-\cos\beta L - \cosh\beta L) = 0 \qquad (I)$$

Rearranging Equations (H) and (I) yields

$$C_1(-\cos\beta L - \cosh\beta L) + C_2(-\sin\beta L - \sinh\beta L) = 0 \qquad (J)$$

$$C_1(sin\,\beta L - sinh\,\beta L) + C_2(-cos\,\beta L - cosh\,\beta L) = 0 \qquad (K)$$

For non trivial solution of (C_1) and (C_2) the determinant of their coefficients must be zero-that is,

$$\begin{vmatrix} -\cos\beta L - \cosh\beta L & -\sin\beta L - \sinh\beta L \\ \sin\beta L - \sinh\beta L & -\cos\beta L - \cosh\beta L \end{vmatrix} = 0 \qquad (L)$$

Expanding the determinant *(L)* gives

$$\cos^2\beta L + 2(\cos\beta L)(\cosh\beta L) + \sin^2\beta L - \sinh^2\beta L + \cosh^2\beta L = 0$$

Further simplification gives

$$(\cos\beta L)(\cosh\beta L) = -1 \qquad (M)$$

Equation $(M)$is the frequency equation which can be rewritten as

$$(\cos\beta_n L)(\cosh\beta_n L) = -1 \qquad (N)$$

The roots of Equation (N), $\beta_n L$, give the natural frequencies of vibration

$$\omega_{n=}(\beta_n L)^2\sqrt{\frac{EI}{\rho AL^4}} = \left(\beta_n{}^2\right)\sqrt{\frac{EI}{\rho A}}\ , n = 1,2,3,\ldots \qquad (O)$$

Where the values of $(B_n L), n = 1,2\ldots$ satisfying Equation (N) for the first four natural frequencies are

$$\beta_1 L = 1.875104$$

$$\beta_2 L = 4.694091$$

$$\beta_3 L = 7.854757 \qquad (5.25)$$

$$\beta_4 L = 10.995541$$

To find the natural frequencies of a straight mast Equations (5.22), (5.23), (O) and (5.25) will be used. The modulus of elasticity *E*, and density ρ of aluminum are

$$E_{AL} = 7.515 * 10^{10} \left({}^{N}/_{m^2}\right)$$

$$\rho_{AL} = 7827.099 \left({}^{Kg}/_{m^3}\right)$$

The mast cross sectional area is

$$A = (\pi/4)[(0.3)^2 - (0.28)^2] = 9.111 * 10^{-3} \quad (m^2) \qquad (5.26)$$

The mast moment of inertia about the y-axis is

$$I = \frac{\pi[(0.3)^4 - (0.28)^4]}{64} = 9.589 * 10^{-5} \ (m^4) \qquad (5.27)$$

Finally using Equation (5.25) and Equation *(O)* the first four natural frequencies are calculated as shown in Table 5.1

Straight mast OD = 0.3m ID = 0.28m	Natural frequency (rad/s)			
	ω_1	ω_2	ω_3	ω_4
	1.7883	11.2072	31.3805	61.4934

Table 5.1 the first four calculated natural frequencies of the straight mast.

5.2 Natural frequencies of a straight mast by SAP2000

The same steps explained and carried out in section (4.1), in building the model geometry and its restraints (steps A1 though A4), are applicable for modal analysis of a straight mast by SAP2000 to find its natural frequencies. Also 4000 divided frame elements will be used (section 4.1, step B3) in building the model for the modal analysis. No loads are applied in this analysis. To perform the modal analysis the following parameters need to be set:

1. Eigenvector modal analysis will be used to determine the un-damped free-vibration mode shapes and natural frequencies of the structure. The analysis case name is (ST. MODAL) as shown in Figure 5.2

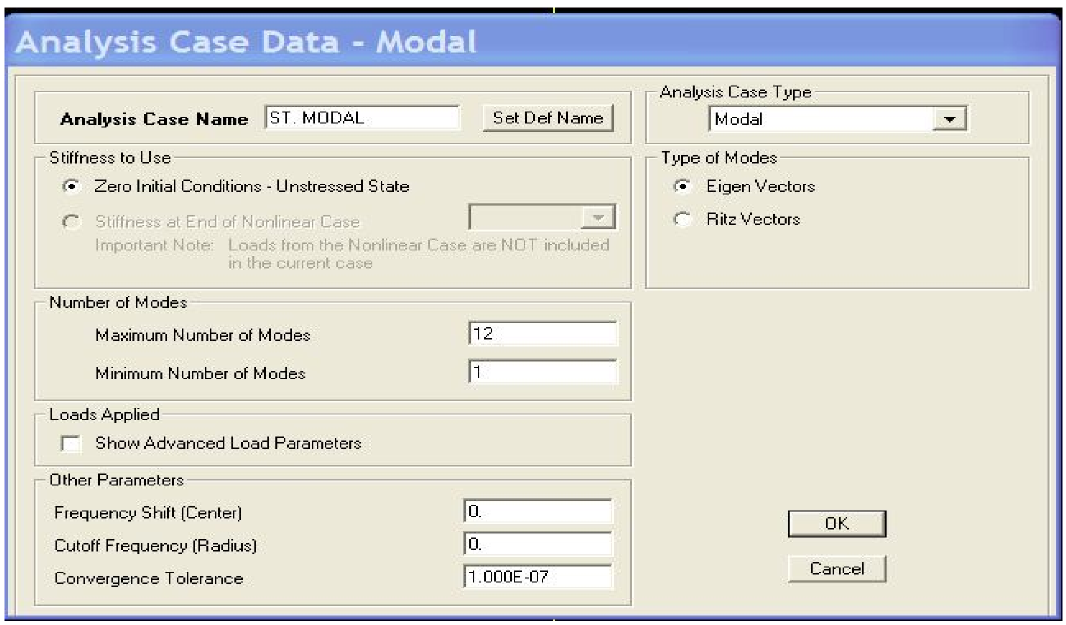

Figure 5.2 Defining the parameters of Eigenvector Modal Analysis.

Eigenvector analysis involves the solution of the generalized Eigenvalue problem

$$\left|\boldsymbol{K} - \Omega^2 \bar{\boldsymbol{M}}\right| \boldsymbol{\phi} = \mathbf{0} \qquad (\mathbf{5.28})$$

Where, $\boldsymbol{K}$ is the stiffness matrix.

$\bar{\boldsymbol{M}}$ is the diagonal mass matrix.

Ω^2 is the diagonal matrix of Eigenvalues.

$\boldsymbol{\phi}$ is the matrix of corresponding Eigenvectors (mode shapes).

Each Eigenvalue-Eigenvector pair is called a natural vibration mode of the structure. The Eigenvalue is the square of the circular frequency (natural frequency), ω, of that mode (unless a frequency shift is used). The cyclic frequency, f, and period, $\tilde{T}$, of the mode are related to ω by:

$$\tilde{\boldsymbol{T}} = \frac{\mathbf{1}}{\boldsymbol{f}} \qquad \boldsymbol{and} \qquad \boldsymbol{f} = \frac{\boldsymbol{\omega}}{\mathbf{2}\boldsymbol{\pi}} \qquad (\mathbf{5.29})$$

2. Choose the stiffness matrix of the unstressed structure (zero initial condition) to solve for the vibration modes as shown in Figure 5.2.
3. Specify the minimum (1) and maximum (12) numbers of modes to be found as shown in Figure 5.2. The program will not calculate fewer than the minimum number of modes, unless there are fewer mass degrees of freedom in the model or if the number of modes in a specified frequency range is less than the minimum number of modes required.
4. Loads are not applied in the Eigenvector modal analysis, since the modes are properties of the structure, not the loading.
5. To seek the vibration modes in a restricted frequency range the following parameters need to be specified:
    a. Frequency shift: the center of the cyclic frequency range.
    b. Cutoff frequency: the radius of the cyclic frequency range. A default value of zero does not restrict the frequency range of the modes.

Modes are found in order of increasing distance of frequency from the frequency shift. This continues until the cutoff frequency is reached, the requested number of modes is found, or the number of mass degrees of freedom is reached. When using a frequency shift, the stiffness matrix is modified by subtracting from it the mass matrix multiplied by ${\omega_o}^2$,

Where,

$$\boldsymbol{\omega_0 = 2\pi * (Frequency\ shift)}$$

The circular frequency, ω, of a vibration mode is determined from the Eigenvalue relative to the frequency shift, $\bar{\mu}$, as:

$$\boldsymbol{\omega = \sqrt{\bar{\mu} + {\omega_o}^2}} \qquad \boldsymbol{(5.30)}$$

For the straight mast under study the frequency shift and the cutoff frequency are both set to value of zero as shown in Figure (5.2).

6. Specify the value of convergence tolerance. SAP2000 solves for the Eigenvalue-Eigenvector pairs using an accelerated subspace iteration algorithm. During the solution phase, the program gives the approximate Eigenvalue after each iteration. As the Eigenvectors converge they are removed from the subspace, and new approximate vectors are introduced. Convergence tolerance control the solution such that the iteration for a particular mode will continue until the relative change in the Eigenvalue between two successive iterations satisfies the following

$$\boldsymbol{\frac{1}{2}\left|\frac{\bar{\mu}_{k+1} - \bar{\mu}_k}{\bar{\mu}_{k+1}}\right| \leq convergence\ tolerance} \qquad \boldsymbol{(5.31)}$$

And for the case of zero frequency shifts, the test for convergence becomes

$$\boldsymbol{\left|\frac{\widetilde{T}_{k+1} - \widetilde{T}_k}{\widetilde{T}_{k+1}}\right| \leq converagence\ tolerance}$$

Or, $\mathbf{(5.32)}$

$$\left|\frac{f_{k+1} - f_k}{f_{k+1}}\right| \le \boldsymbol{converagence\ tolerance}$$

Where, $\tilde{T}$ is the cyclic period.

f is cyclic frequency.

k and $k + 1$ are successive iteration numbers.

The default value (1*10^{-7}) will be used for this analysis as shown in Figure 5.2.

Once all previous parameters are set, run the modal analysis. The results are given in the form of “Eigenvalue” values or “Period” values as shown in Figure 5.3. Those values are used to calculate the natural frequencies of the mast (Eqn.5.29) as shown in Table 5.2.

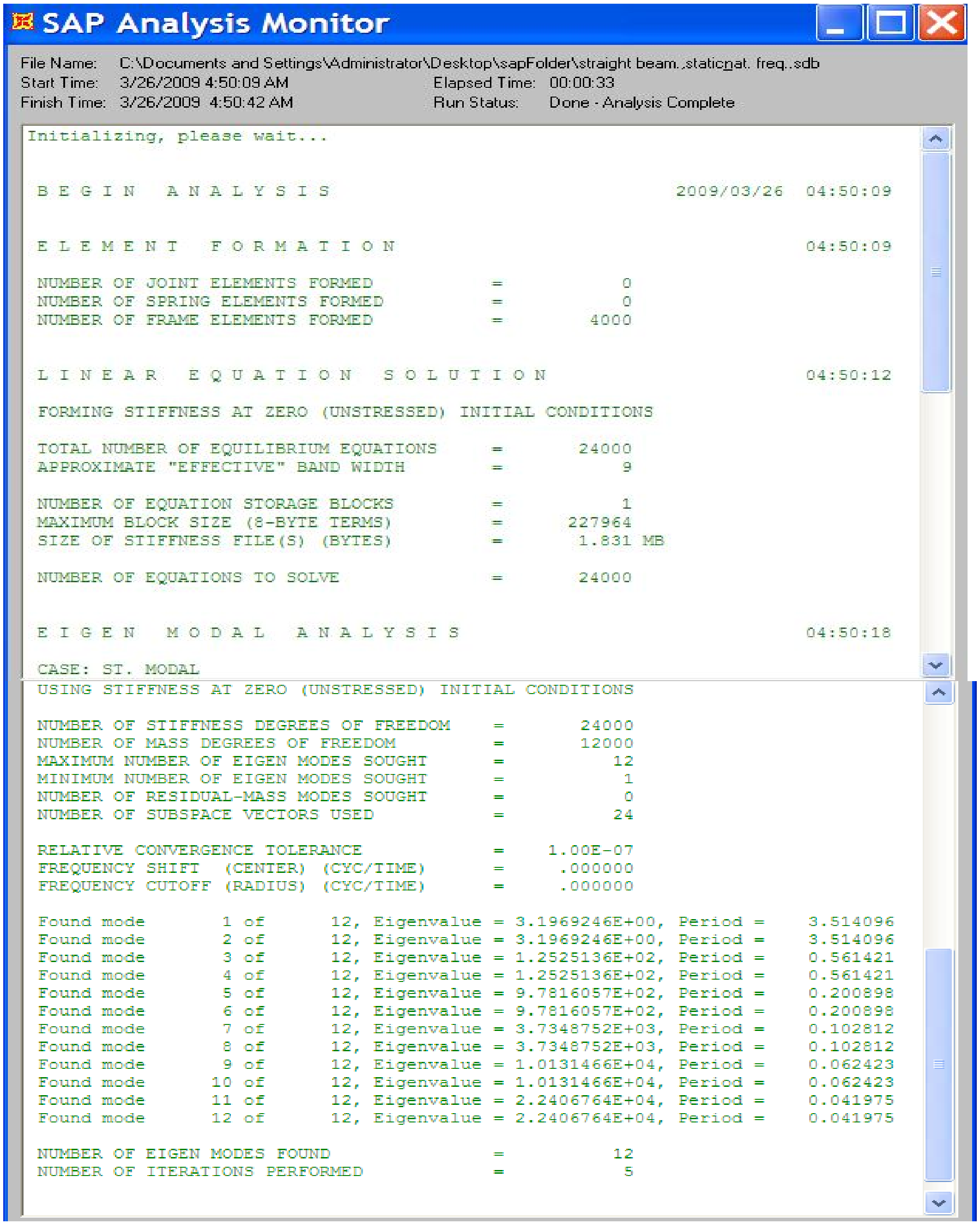

Figure 5.3 Running the Eigenvector modal analysis for the straight mast case.

Straight mast D = 0.3m d = 0.28m		1st mode	2nd mode	3rd mode	4th mode
	Period (s)	3.514096	0.561421	0.200898	0.102812
	Natural frequency (rad/s)	1.788	11.192	31.275	61.1133

Table 5.2 the first four natural frequencies of the straight mast using SAP2000

5.3 Comparison of SAP2000 and close form solution for FEA validation

A comparison of the natural frequency results obtained using close form solution and SAP2000 for the straight mast is shown in Figure 5.3

Straight mast D = 0.3m d = 0.28m	ω_1(rad/s)	ω_2(rad/s)	ω_3(rad/s)	ω_4(rad/s)
SAP2000	1.788	11.192	31.275	61.1133
Closed form solution	1.7883	11.2072	31.3805	61.4934

Table 5.3 A comparison of natural frequency results using SAP and formula for the straight mast.

It is obvious from comparing the two sets of results (SAP2000, and closed form solution), that the difference in values is very small and negligible. This leads to the conclusion that SAP2000 gives very accurate results for natural frequencies.

CHAPTER VI

DYNAMIC RESPONSE OF STRAIGHT AND TAPERED MAST UNDER VORTEX SHEDDING PHENOMENON

6.1 Natural frequencies of a tapered mast by SAP2000

The same steps explained and carried out in sections 4.1 and 4.2 for configuring the tapered mast model geometry and its restraints are applicable for tapered mast modal analysis using SAP2000. A total of 4000 frame elements are used in building the model. Also no loads are applied in the natural frequency modal analysis. Eigenvector method is used, and the parameters of the modal analysis are set exactly the same way as they were explained and carried out in Section 5.2 (steps 1 through 6). The Eigenvalue modal analysis is performed for all three tapered designs (I, II, III), and the first four natural frequencies are found for each case as shown in Table 6.1

Tapered design case #	Period(s)				Natural frequency (rad/s)			
	1st mode	2nd mode	3rd mode	4th mode	ω_1	ω_2	ω_3	ω_4
I	2.398056	0.50580	0.19700	0.103337	2.6201	12.422	31.894	60.8028
II	1.691734	0.47144	0.20340	0.110835	3.7140	13.327	30.890	56.6895
III	1.344889	0.44886	0.21521	0.123680	4.6719	13.997	29.194	50.8019

Table 6.1 The first four natural frequencies of the three tapered designs using SAP2000

While moving from design (I) to design (III), the mass of the tapered mast is allocated and moved closer to the fixed end. The mass distribution of the tapered configuration increases the overall stiffness of the mast. This increase is more pronounced moving from tapered shape of I to that of III, and increased the first two natural frequencies as shown in Table 6.1.

6.2 Dynamic forced-response of straight and tapered masts under periodic vortex shedding excitations, SAP solution

The same steps explained and carried out in sections (4.1) and (4.2), to build the straight and tapered masts model geometry and its restraint, are applicable for the dynamic forced-response analysis. A total of 4000 frame elements are used in this analysis. Time-History analysis in SAP2000 is used to find the dynamic response of the mast under the periodic vortex shedding excitations. It is a step-by-step analysis that involves solving the dynamic equilibrium equations given by:

$$\boldsymbol{K}\,\boldsymbol{u}(\boldsymbol{t}) + \boldsymbol{O}\dot{\boldsymbol{u}}(\boldsymbol{t}) + \bar{\boldsymbol{M}}\ddot{\boldsymbol{u}}(\boldsymbol{t}) = \boldsymbol{r}(\boldsymbol{t}) \qquad (\mathbf{6.1})$$

Where, $\boldsymbol{K}$ is the stiffness matrix.

$\boldsymbol{O}$ is the proportional damping matrix.

$\overline{\boldsymbol{M}}$ is the diagonal mass matrix.

$\boldsymbol{u}$ is the relative displacements with respects to the ground.

$\dot{\boldsymbol{u}}$ is the relative velocities with respect to the ground

$\ddot{\boldsymbol{u}}$ is the relative accelerations with respect to the ground.

$\boldsymbol{r}$ is the vector of applied load.

$\boldsymbol{t}$ is time.

The load, $r(t)$, applied in a given Time-History analysis may be an arbitrary function of space and time. It can be written as a finite sum of spatial load vectors,P_i, multiplied by time function, $\tilde{f}_i(t)$, as:

$$\boldsymbol{r}(\boldsymbol{t}) = \sum_{\boldsymbol{i}} \tilde{\boldsymbol{f}}_{\boldsymbol{i}}(\boldsymbol{t})\boldsymbol{P}_{\boldsymbol{i}} \qquad (6.2)$$

Where, $\tilde{\boldsymbol{f}}_{\boldsymbol{i}}(\boldsymbol{t})$, is the i-th time function, varying along the length of the mast, of the vector of applied loads.

SAP2000 uses load case and/or acceleration loads to represent the spatial load vectors. The time functions can be arbitrary functions of time or periodic functions.

The load $r(t)$ represents the harmonically varying lift force $F_l(t)$ given by Equation (1.4) in chapter one:

$$\boldsymbol{r}(\boldsymbol{t}) = \boldsymbol{F}_{\boldsymbol{l}}(\boldsymbol{t}) = \frac{\mathbf{1}}{\mathbf{2}}\, \boldsymbol{C}_{\boldsymbol{l}}\, \boldsymbol{\rho}_{\boldsymbol{air}}\, \boldsymbol{U}^{\mathbf{2}}\boldsymbol{A}_{\boldsymbol{p}} \sin \overline{\boldsymbol{\omega}}\boldsymbol{t} \qquad (6.3)$$

Where, $\boldsymbol{C_l}$ is the fluctuating lift coefficient ($C_l = 1$ for circular cylinder).

$\boldsymbol{\rho_{air}}$ is the density of the flowing air (ρ = 1.2 kg/m³ at standard atmospheric pressure and 20 C°)

$\boldsymbol{U}$ is the velocity of flowing air (m/s)

$\boldsymbol{A_p}$ is the projected area of the circular cylinder perpendicular to the direction of U (m²).

$\boldsymbol{\bar{\omega}}$ is the circular frequency of vortex shedding (rad/s).

$\boldsymbol{t}$ is the time (s).

Since the projected area is a rectangular shape with a length equals to the masts length, and a width equals the outside diameter, D, for straight mast, or average of the outside diameters ; the top and bottom, $\bar{D}$, for tapered mast. This gives:

$$\boldsymbol{A_p = 25 * 0.3 = 7.5}\ (\mathrm{m}^2)$$

The circular frequency of vortex shedding $\boldsymbol{\bar{\omega}}$ is given by

$$\boldsymbol{\bar{\omega} = 2\pi f_s} \qquad (\boldsymbol{a})$$

Where, $\boldsymbol{f_s}$ is the frequency of vortex shedding in Hertz.

It is important to note that the value of f_s is related to Strouhal number by Equation (1.2) in chapter one as

$$\boldsymbol{St = \frac{f_s \bar{D}}{U}} \qquad (\boldsymbol{b})$$

Where $\boldsymbol{\bar{D}}$ is the average outside diameter of the mast(m). The value of f_s is calculated for different values of velocities. The velocity values chosen should result in Reynolds number values that still fall in ranges of constant Strouhal number; $St = 0.21$ for low speeds and $300 < Re < 2 * 10^5$ or $St = 0.27$ for high speeds and $Re > 3.5 * 10^6$. It is worth noting that in the previous two stable Reynolds number ranges vortex shedding is of a regular pattern, and occurs with a well defined frequency f_s **.** Summarizing the above gives:

$$\mathbf{0.21} = \frac{f_s\,(\mathbf{0.3})}{U}\ \boldsymbol{for}\ \ \mathbf{300} < Re < 2 * \mathbf{10^5}$$

$$(\mathbf{6.4})$$

$$\mathbf{0.27} = \frac{f_s\,(\mathbf{0.3})}{U}\ \boldsymbol{for}\ \ \boldsymbol{Re} > 3.5 * \mathbf{10^6}$$

Now the values of velocity should fall in two ranges, whose limits are decided by the value of Reynolds number. In other words, three Reynolds number values $(300, 2 * 10^5,$ and $3.5 * 10^6)$ are used to calculate the limits of two speed ranges. Equation (1.1) in chapter one, which defines Reynolds number, is used for this purpose

$$\boldsymbol{Re} = \frac{\boldsymbol{U\bar{D}}}{\boldsymbol{\nu}_{\mathbf{air}}} \qquad (\boldsymbol{c})$$

Where, $\boldsymbol{\nu}_{\mathbf{air}}$ is the kinematic viscosity of flowing air ($\nu = 1.51 * 10^5\ ^{m^2}/_{s}$ at standard atmospheric pressure and 20 C^o)

$\boldsymbol{\bar{D}}$ is the average outside diameter of the mast ($D = 0.3$m)

This gives two velocity ranges

$$\mathbf{0.015} < U < 10.10\ \ for\ 300 < Re < 2 * \mathbf{10^5}$$

$$(\mathbf{6.5})$$

$$\boldsymbol{U} > 176.20\ \ \ for\ \ Re > 3.5 * \mathbf{10^6}$$

The roughness of the terrain retards the wind near the ground. The lower layers of air then retard those above them, resulting in different wind speeds from ground level until the retarding forces are diminished to zero. At that height, called the gradient height, changes in wind speed are unaffected by ground effects. Taking this into account, the wind velocity profile is assumed to be of triangular shape with zero velocity at base of the mast and maximum velocity at the top. Also it is assumed that the length of the mast is the gradient height [5]. The velocity profile represents the spatial load vectors, P_i, distribution along the length of the mast. For a given velocity, U, the time function $f_i(t)$ is constant along the length of the mast. This gives the following

$$\boldsymbol{r(t) = F_l(t) = \check{f}(t)\sum_{i} P_i(z_i) = \frac{1}{2}\, C_l \rho_{air} U^2 A_p \sin \bar{\omega} t} \qquad \textbf{(6.6)}$$

Where, $\boldsymbol{\check{f}(t)}$ is the constant, along the length of the mast, time function of the vector of applied loads.

Therefore the spatial distribution for a given wind velocity , U, is

$$\boldsymbol{\sum_{i} P_i(z_i) = \frac{1}{2}\, C_l \rho_{air}\, U^2 A_p} \qquad \textbf{(6.7)}$$

Where, $\boldsymbol{P_i}$ is the i-th spatial load vector of the vector of applied loads.

$\boldsymbol{z_i}$ is the location of the i-th spatial load vector**,** P_i , in the z-direction along the mast’s longitudinal axis.

Twenty concentrated loads vectors acting in the positive x-direction along the length of the mast at 1.25 meters increment represents the spatial load distribution. The value of each concentrated load is given by

$$\boldsymbol{P_i(z_i)} = \left(\frac{\boldsymbol{P_{20} - P_0}}{\boldsymbol{z_{20} - z_0}}\right)\boldsymbol{z_i} = \left(\frac{\boldsymbol{P_{20}}}{\boldsymbol{25}}\right)\boldsymbol{z_i} \qquad (\mathbf{6.8})$$

Where, $\boldsymbol{P_0}$ is the concentrated force at base ($P_0 = 0$).

$\boldsymbol{P_{20}}$ is the concentrated force or the 20-th spatial load vector at the free end of the mast, of the vector of applied loads, and $z_{20} = 25$

$\boldsymbol{z_i}$ is the location of the i-th concentrated force (i-th load vector) along the mast longitudinal axis (z -axis) [$z_i = z_{i-1} + 1.25$ and $z_0 = 0, \ \mathrm{i} = 1,2,\ldots,20$].

Combining Equations (6.7) and Equation (6.8) for a given flow velocity, U , gives

$$\boldsymbol{\frac{1}{2} C_l \rho_{air} U^2 A_p} = \sum_{i=1}^{i=20} \boldsymbol{P_i(z_i)} = \sum_{i=1}^{i=20} \left(\frac{\boldsymbol{P_{20}}}{\boldsymbol{25}}\right)\boldsymbol{z_i} = \left(\frac{\boldsymbol{P_{20}}}{\boldsymbol{25}}\right)\sum_{i=1}^{i=20} \boldsymbol{z_i} = \left(\frac{\mathbf{262.5}}{\mathbf{25}}\right)\boldsymbol{P_{20}} \ (\mathbf{6.9})$$

And that gives

$$\boldsymbol{P_{20}} = \left(\frac{\mathbf{25}}{\mathbf{262.5}}\right)\left(\frac{\mathbf{1}}{\mathbf{2}}\right)(\mathbf{1})(\mathbf{1.2})(\mathbf{7.5})\boldsymbol{U^2} = \mathbf{0.4286}\,\boldsymbol{U^2} \qquad (\mathbf{6.10})$$

Substituting Equation (6.10) into (6.8) gives

$$\boldsymbol{P_i(z_i)} = \mathbf{0.017143}\,\boldsymbol{U^2 z_i} \qquad (\mathbf{6.11})$$

Step (B1) in chapter four is used to define the Time-History load case “20 Const”. For a given speed, twenty concentrated loads, taken at 1.25 meters increments, are defined under load case “20 const” using Equation (6.11) as shown in Figure 6.1.

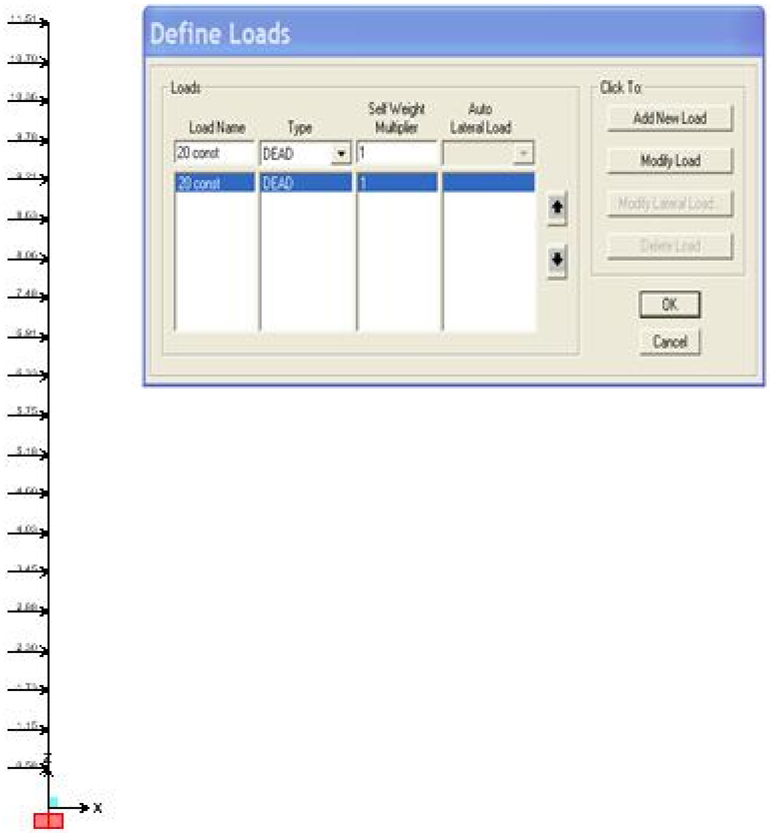

Figure 6.1 Defining load case “20 const “for the Time-History analysis, and its twenty concentrated loads for a certain speed.

The dynamic response of the mast is studied for four different wind velocities, namely 2, 4, 6, and 10 (m/s). Using Equations (a), (6.4), (6.5), (6.6), and (6.11) to calculate the lift force $F_l(t)$ for all four values of speed (2, 4, 6, and 10) respectively gives

$$\boldsymbol{F_{1l}(t)} = \left(\sum_{i=1}^{i=20} \mathbf{0.06857}\boldsymbol{z_i}\right) \mathbf{sin\,8.8}\boldsymbol{t} \qquad (\mathbf{6.11.}\boldsymbol{a})$$

$$\boldsymbol{F_{2l}(t)} = \left(\sum_{i=1}^{i=20} \mathbf{0.27427}\boldsymbol{z_i}\right) \mathbf{sin\,17.6}\boldsymbol{t} \qquad (\mathbf{6.11.}\boldsymbol{b})$$

$$\boldsymbol{F_{3l}(t)} = \left(\sum_{i=1}^{i=20} \mathbf{0.61704}\boldsymbol{z_i}\right) \mathbf{sin\,26.4}\boldsymbol{t} \qquad (\mathbf{6.11.}\boldsymbol{c})$$

$$\boldsymbol{F_{4l}(t)} = \left(\sum_{i=1}^{i=20} \mathbf{1.714}\boldsymbol{z_i}\right) \mathbf{sin\,44}\boldsymbol{t} \qquad (\mathbf{6.11.}\boldsymbol{d})$$

Where, $F_{sl}(t)$ is the s-th lift force corresponding to a certain value of speed (2, 4, 6, and 10 m/s).

Using Equations (6.11.a) through (6.11.d), four different constant, along the length of mast, time functions, $\check{f}_s(t)$, related to four values of velocities, are defined in SAP2000 using the "Time history sine function definition" as shown in Figure 6.2

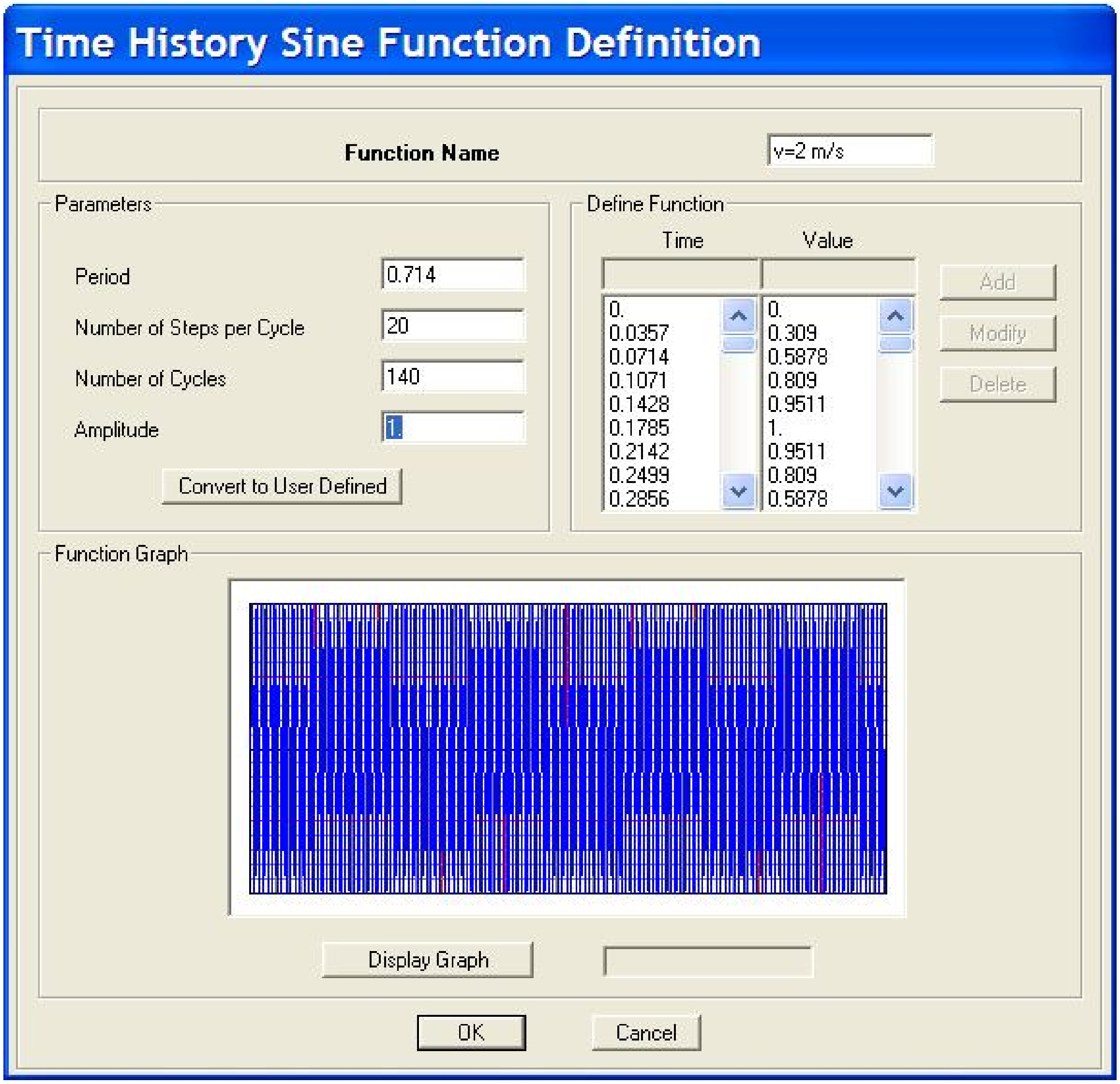

Figure 6.2 Defining the time history sine function of the Time-History analysis for different values of wind velocity.

Each time function is given a specific name ($V = 2\,\mathrm{m/s}$, $V = 4\,\mathrm{m/s}$, … ,$V = 10\,\mathrm{m/s}$) as shown in Figure 6.2. Four parameters are defined for each time history sine function. The first parameter is “period”, which is given by

$$\boldsymbol{\tau_s} = \frac{\mathbf{2\pi}}{\boldsymbol{\bar{\omega}_s}} \qquad (\mathbf{6.12})$$

Where, $\boldsymbol{\tau_s}$ is the period of the s-th time function $\check{f}(t)$.

$\bar{\boldsymbol{\omega}}_{\boldsymbol{i}}$ is the circular frequency of vortex shedding for the s-th constant time function $\check{f}(t)$.

The second parameter is "number of steps per cycle", which is the number of steps (i.e., function value points) provided for each cycle of the function. 20 steps are chosen for the sine time history function as shows in Figure 6.2.

The third parameter is "number of cycles"; it is defined such that when multiplied by the "Period" parameter gives a total of one hundred seconds. The total one hundred seconds represent the time span over which sine time history function and Time-History analysis are defined. The last parameter is "Amplitude" which is the maximum function value of the sine function. A value of one is assigned for this parameter as shown in Figure 6.2. Table 6.2 below gives the time function parameters for the four time functions defined by Equation (6.11).

Time function Name	Period	Number of steps per cycle	Number of cycle	Amplitude
V = 2m/s	0.7143	20	141	1
V = 4m/s	0.3571	20	281	1
V = 6m/s	0.2381	20	421	1
V = 10m/s	0.1429	20	701	1

Table 6.2 Defining the parameters of the time history sine functions for all four speeds

Eigenvector modal analysis by SAP2000 (sections 5.2, and 6.1) is performed as a mandatory previous step before running Time-History analysis. Figure 6.3 shows the necessary steps to perform for the Time-History analysis.

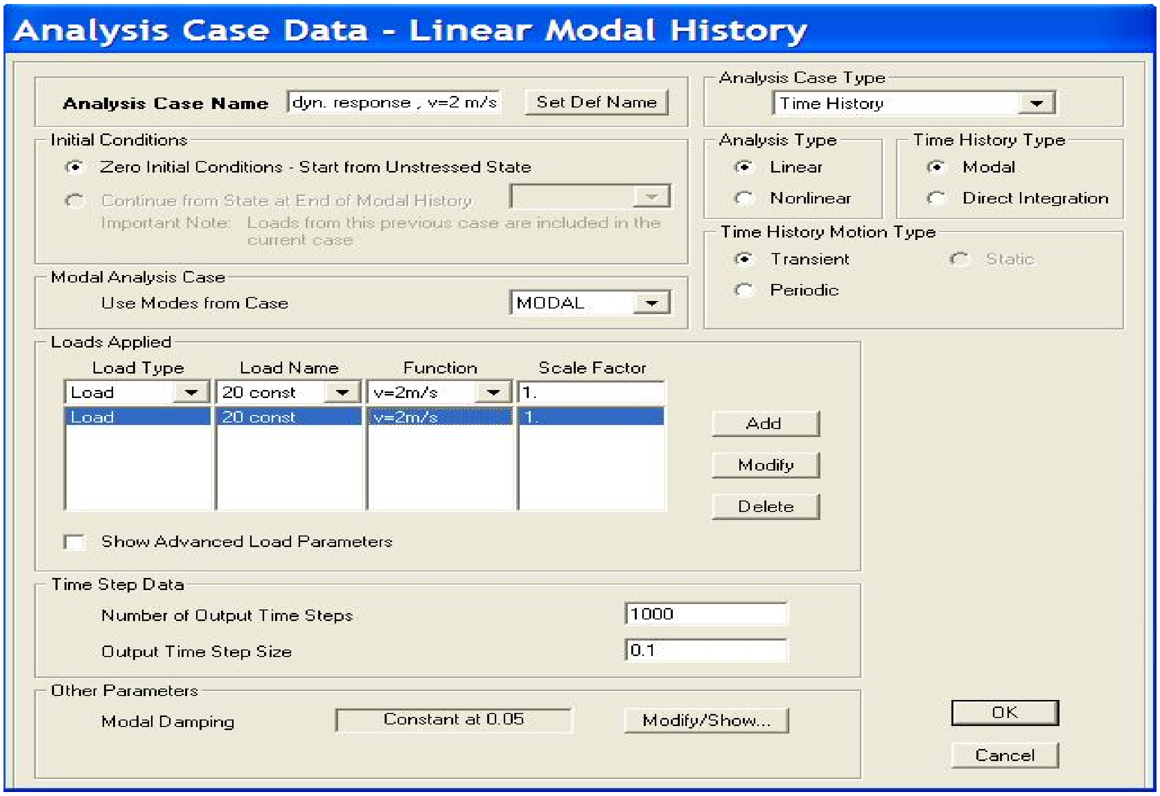

Figure 6.3 Defining the parameters of the Linear Modal History analysis used to find dynamic forced-response of straight and tapered masts under periodic vortex shedding excitations.

The steps carried out for Time-History analysis are:

1. Choose option “Linear” under “Analysis Type” parameter. The “Linear” choice implies the following:

    a. Structural properties (stiffness, damping, etc) are constant during the analysis.

    b. The analysis starts with zero stress. It doesn’t contain loads from any previous analysis, even if it uses the stiffness from a previous non linear analysis.

c. All displacements, stresses, reactions, etc., are directly proportional to the magnitude of the applied loads. Also the results of different linear analyses may be superposed.

2. Choose option “Modal” under “Time-History Type” parameter as shown in Figure 6.3. Modal superposition method of response analysis is used to solve the dynamic equilibrium equations of motion for the complete structure. It involves closed-form integration of modal equations to compute the response. Also it is worth noting that Linear Modal Time-History analysis is usually more accurate and efficient than Linear Direct Integration Time-History analysis, which involves the direct integration of the full equations of motion of the whole structure, without the use of modal superposition, at each time step. Direct Integration method is used with problems involving impact and wave propagation that might excite a large number of modes which is not the case in our analysis.

3. Chose option “Transient” under “Time History Motion Type” parameter. Transient option represents the usual case, where the structure starts at rest and is subjected to the specified loads only during the time period specified for the analysis, and that means the applied loads are considered as a onetime event, with a beginning and end. It is worth noting that the “Periodic” option considers the applied loads to be periodic, that is, they repeat indefinitely with a period given by the length of the analysis. Also the “Periodic” option considers the response to be periodic; the program adjusts the displacement and velocity at the beginning of

the analysis to be equal to those values at the end of the analysis. The result under "Periodic" option the response is steady state response to a periodically applied load of arbitrary time-variation with all transient response damped out.

4. Choose option "Zero Initial Condition" under "Initial Conditions" parameter. For linear transient analysis, which is the case, zero initial conditions are always assumed. Initial conditions describe the state of the structure the beginning of a time-history case. These include:
   - Displacement and velocities.
   - Internal forces and stress.
   - Energy values for the structure.
   - External loads.

5. Chose "MODAL" under "Modal Analysis Case" parameter. Modal Time-History analysis is based on superposition. Select the name of the modal analysis case, "MODAL", whose modes are the basis for the Time-History analysis.

6. Under "Load Applied" parameter, choose the "Load" option under "Load Type", because the load applied is a load case. Choose load case "20 Const" under "Load Name", which represents the spatial distribution of the twenty concentrated loads related to a specific speed. Under "Function" the option $(V = 2\,\mathrm{m/s}, V = 4\,\mathrm{m/s}, \ldots, V = 10\,\mathrm{m/s})$ is selected. It represents the time function used for

specific speed. Also choose a scale factor of value one for the analysis. Scale factor is a load multiplier.

7. Under “Time Step Data” choose one thousand for the ‘Number of Out Put Time Steps” parameter, and (0.1) value for “Output Time Step Size” parameter. The time span over which the analysis is carried out is given by the multiplication of the previous two parameters, and that is one hundred seconds. Response is calculated and saved at time zero and the subsequent output time steps, although the analysis will compute intermediate response values at every time step of every applied load time history function in order to accurately capture the full effect of the loading.

8. Choose a value of (0.05) for “Modal Damping” parameter. The damping in the structure is modeled using modal damping. Each mode has a damping ratio, which is measured as a fraction of critical damping and must satisfy:

$$\mathbf{0} \leq \boldsymbol{damping\ ratio} < 1$$

For a Linear Modal Time-History analysis case, the modal damping ratio can be specified as:

- Constant for all modes (a value of 0.05 is chosen for this dynamic analysis).
- Linearly interpolated by period or frequency. By specifying the damping ratio at a series of frequency or period points, and between the specified points the damping is linearly interpolated. Outside the specified range,

the damping ratio is constant at the value given for the closest specified point.

- Mass and stiffness proportional. This mimics the proportional damping, except that the damping value is never allowed to exceed unity.

Finally the Linear Transient Time-History analysis is run as shown in Figure 6.4.

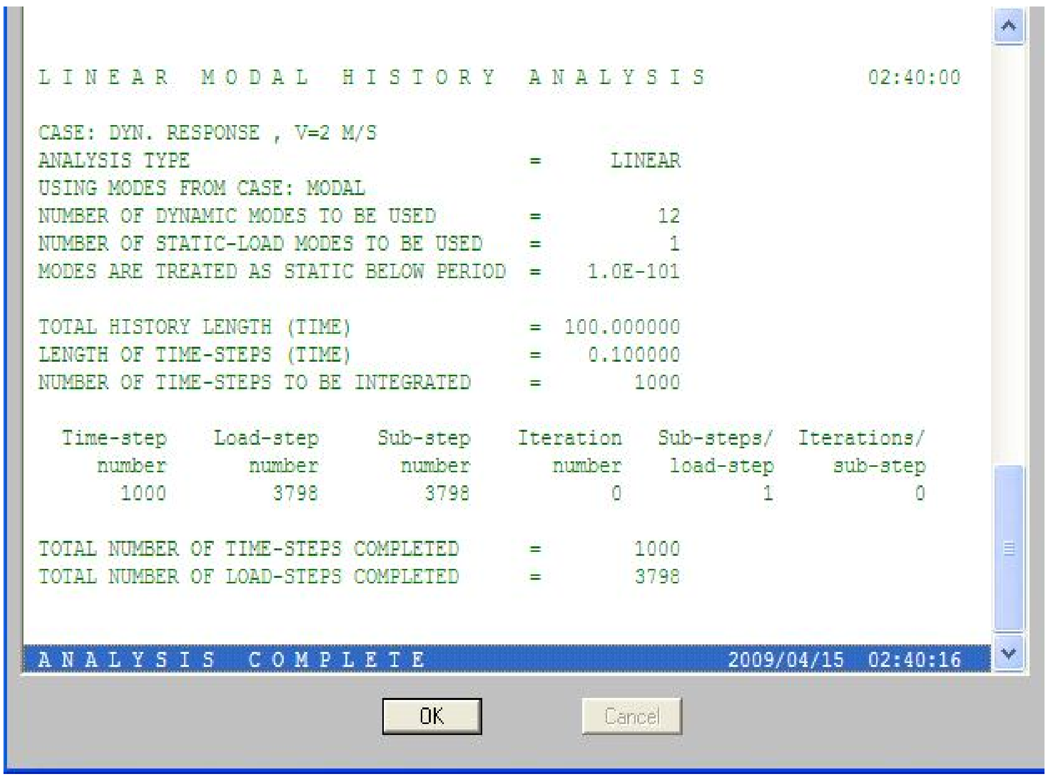

Figure 6.4 Running the Linear Transient Modal Time-History analysis.

The dynamic response of the tip of the straight mast for a velocity of 6 m/s is shown in Figure 6.5.a. it shows the total response(transient, and steady state) over the rated analysis time span (100 seconds). Figure 6.5.b shows the steady state part of the response over a small range of time, from 50 second to 60 seconds, of the total time span of the analysis. The fluctuation of the steady states response between maximum and minimum values is also apparent in Figure 6.5.b. The rest of the results are displayed in appendix (B).

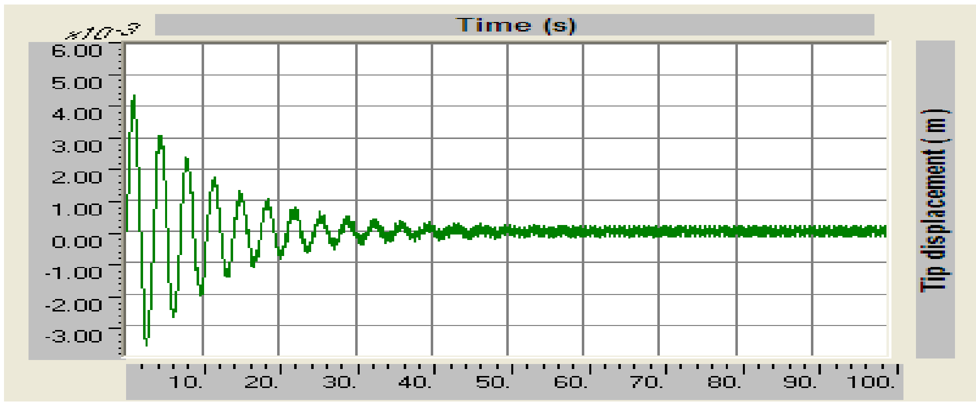

Figure 6.5.a The total dynamic response of the tip of the straight mast with wind velocity of 6m/s over a time span of 100 seconds.

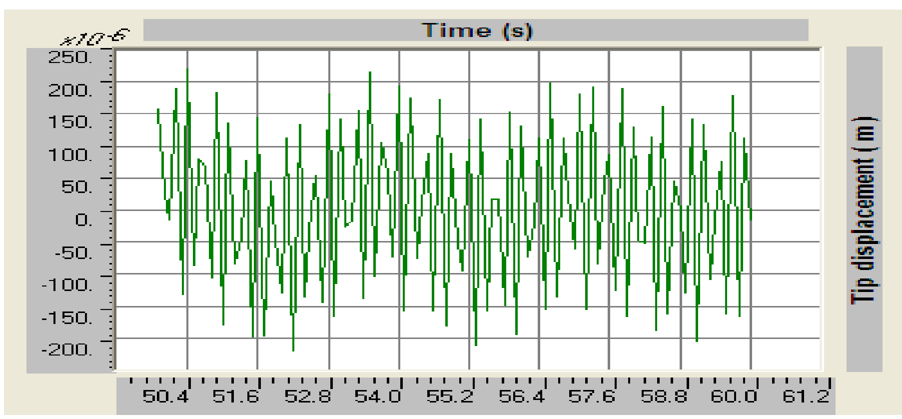

Figure 6.5.b Part of the steady state dynamic response of the tip of the straight mast with wind velocity of 6m/s taken between 50 and 60 seconds.

6.3 Comparing results of dynamic response for straight and tapered masts

The dynamic response of the straight and three proposed tapered designs (I, II, III) for the four values of velocity is performed. The amplitude of the steady states dynamic response of the four mast designs, and at four values of velocity is shown in Table 6.3. the amplitude data were taken between 50 and 60 seconds.

Case#	Steady State Response Amplitude ($* 10^{-6}$ m)			
	V = 2 m/s	V = 4 m/s	V = 6 m/s	V = 10 m/s
Straight	320	240	200	350
I	400	400	270	480
II	600	900	340	950
III	800	2250	2000	1300

Table 6.3 The amplitude of the steady state response of all four mast designs for different values of velocity taken between 50 and 60 seconds.

The values in Table 6.3 are represented graphically by curves of amplitude versus speed for all four designs as shown in Figure 6.6

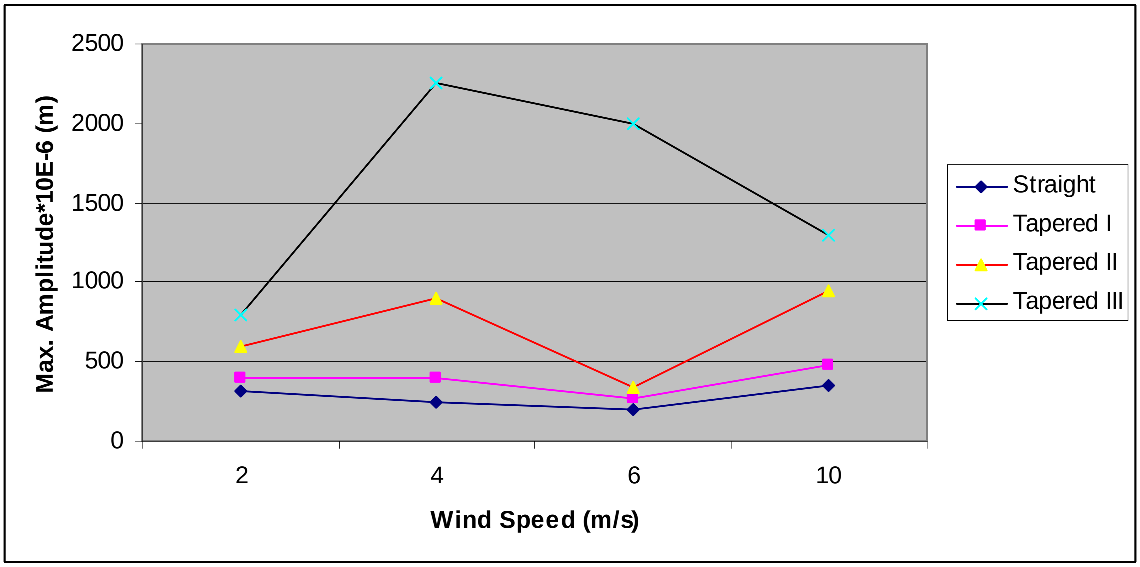

Figure 6.6 The maximum amplitude of steady state response for all four mast designs at different values of wind speed.

6.4 Use of Strouhal number for calculation of critical wind speed for straight and tapered masts

Critical wind speed is the minimum velocity of wind at which the frequency of vortex shedding; frequency of lift force $F_l(t)$, equals one of the natural frequencies of the mast. Recalling Equation (1.2) in chapter one,

$$\boldsymbol{St} = \frac{\boldsymbol{f_s \bar{D}}}{\boldsymbol{U}} \qquad (\boldsymbol{c})$$

and

$$\boldsymbol{f_s} = \frac{\boldsymbol{\bar{\omega}}}{\boldsymbol{2\pi}} \qquad (\boldsymbol{d})$$

Take the value of St to be equal to (0.21), and equate the vortex shedding frequency, f_s, to the natural frequency of the mast, f. Substitute a value of (0.3) for the outside mast diameter, $\overline{D}$, and finally combine Equations (c) and (d). This gives

$$\boldsymbol{U = U_{cr} = \frac{(0.3)}{(0.21)(2\pi)}\,\omega = 0.2274\omega} \quad \mathbf{(6.13)}$$

Where, $\boldsymbol{U_{cr}}$ is the critical wind speed (m/s).

$\boldsymbol{\omega}$ is the natural frequency of the mast (rad/s).

The critical wind speed is calculated, using Equation (6.13), for the first and second natural frequencies of straight and tapered designs (I, II, III). Natural frequency values, $\boldsymbol{\omega}$, are taken from Table 5.2 in chapter five, and Table 6.1 in chapter six. Table 6.4 shows all the critical wind speed calculations. Table 6.4 sums all critical wind calculations, and Figure 6.7 shows them graphically.

Design	Critical wind speed U_{cr}(m/s)	
	First natural frequency	Second natural frequency
Straight	0.407	2.544
I	0.596	2.824
II	0.844	3.030
III	1.062	3.183

Table 6.4 Critical wind speed,U_{cr}, for first and second natural frequencies of mast four designs.

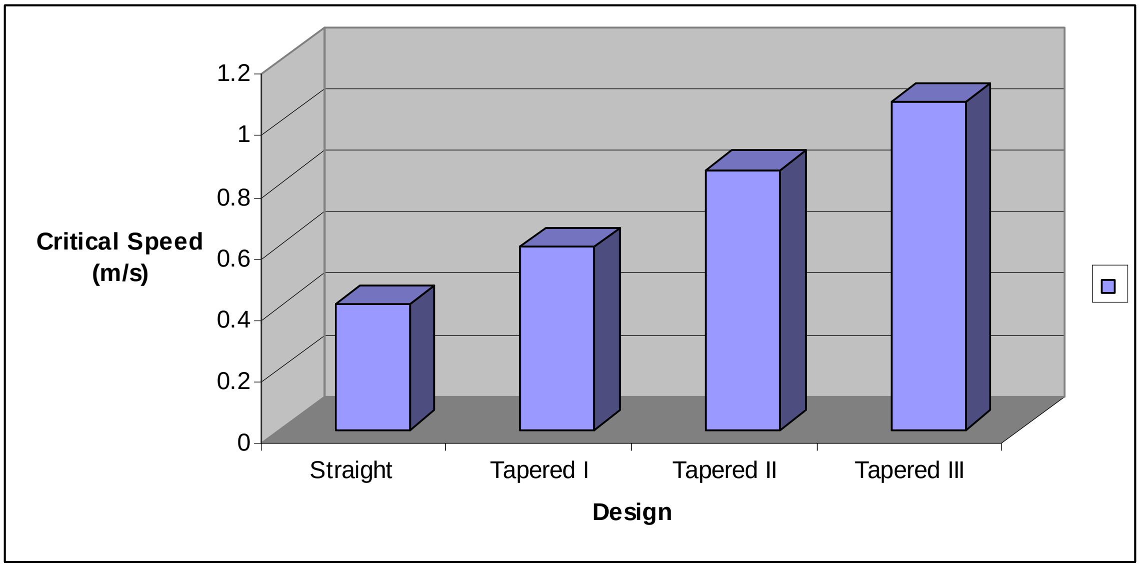

Figure 6.7a Critical wind speed, U_{cr}, for the first natural frequency of all four mast designs.

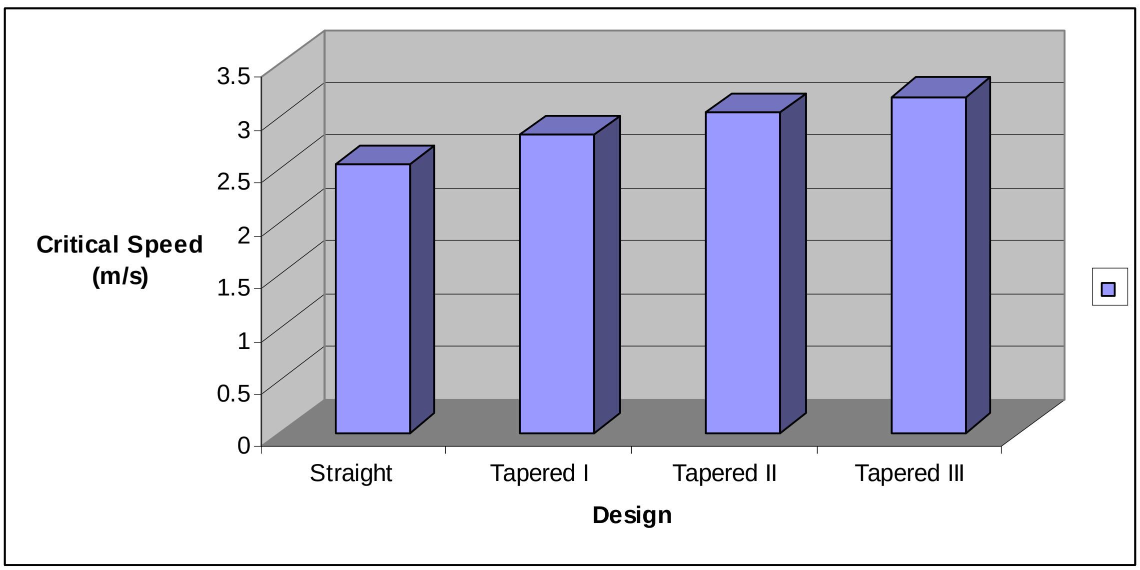

Figure 6.7.b Critical wind speed, U_{cr}, for the second natural frequency of all four mast designs.

It is obvious from Figure 6.7 that the critical wind speed increased with tapering.

CHAPTER VII

CONCLUSION AND FUTURE WORK

7.1 Discussion and conclusion

This study is to understand how to improve the dynamic behavior of a long slender straight mast subjected to flow induced vibrations that is caused by vortex shedding excitations. To achieve this, 3 different tapered mast configurations were proposed. While maintaining the same mass, length, and thickness as of the straight mast, three tapered designs (I, II, III) with increased tapering angles were introduced and analyzed. The first four natural frequencies of straight and tapered designs were calculated using the SAP2000. Also the dynamic response for all four designs under vortex shedding excitations were studied. Vortex shedding forces components in the cross-wind direction were modeled, using SAP2000, as a harmonically varying force with respect to time. The amplitude of the sinusoidal force was also varying along the length of the mast. The dynamic response for different values of wind velocity was calculated.

The first and second natural frequencies of the mast increased with increasing the tapering angle. This is due to the fact that increasing the tapering increased the stiffness, which consequently increased the natural frequency of the mast. Even though the third and fourth natural frequencies did not follow the same trend, they had limited effects on the dynamic response of the structure; especially the fourth natural frequency. This is due to the fact that frequencies of vortex shedding exciting forces are usually low (< 10Hz). The fourth natural frequency of the straight mast is almost ten Hertz, which explains why it is less significant for vortex shedding excitations compared to the first and second natural frequency. Also the amplitude of the steady state response of the tip of the mast, Figure 6.6, increased for all wind speed values with increased tapering of the mast. Even though, increasing the tapering of the mast resulted in a stiffer mast. The very top of the free end of the mast ended up having less material and consequently less area moment of inertia the reduction in area moment of inertia closer to the tip, and undergoing the same wind speed (lifting forces) resulted in an increase in the amplitude of response with increased tapering.

Increasing the natural frequency of the mast with tapering resulted in increasing the ability of the mast to withstand higher values of velocity before falling under the influence of resonance. In other words tapering increased the critical wind speed, U_{cr}, and consequently increased the structural stability of the mast under vortex shedding excitations.

7.2 Contribution of this work

In this study we introduced SAP2000 as a structural dynamic analysis tool. SAP 2000 is very sophisticated software that can be used for different projects ranging from a simple 2D-static frame analysis to a large complex 3D-nonlinear dynamic analysis. We built a model, using SAP2000, which represents tapered slender mast or beam. This model can be used to find the natural frequencies of tapered masts or beams. It also can be used for dynamic analysis involving vortex shedding excitations. Finally SAP2000 enabled us to model the vortex shedding forces as a function of both time and space, which is a true representation of the actual case.

7.3 Future Work

It is essential to use other FEA software, and compare the numerical results with each other. This will help improving the model presentation of the actual structure.

Throughout this study, our concern was analyzing periodic vortex shedding excitations and their dynamic effect on masts. There is a range of wind velocity over which vortex shedding excitations are random (non periodic), and their dynamic effects need further investigation.

The model built in this study was tapered with uniform wall thickness. A tapered model with non uniform thickness needs to be investigated. Studying the effects of non uniform wall thickness on the natural frequency of the mast, and the impact it will have on the masts dynamic response is essential.

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APPENDICES

APPENDIX A

```
> # The straight mast case.
> # (P) or (F) is the concentrated force at the tip of the mast.
> ro := proc (Z) 15 end;
> ri := proc (Z) 14 end;
> E  :=7.515*(10^6);
> L  := 2500;
> G  :=2.8904*(10^6);
> A := proc (ro,ri) Pi*((ro(Z))^2-(ri(Z))^2) end;
>  Ix := proc (ro,ri) Pi*((ro(Z))^4-(ri(Z))^4)/4 end;
> P=F;
> V=F;
> M := proc (Z) F*(Z-L) end;
> U1 := (M(Z))^2/(2*E*Ix(ro,ri));
> U2 := F^2*(2)/(2*G*A(ro,ri));
> U := int( U1+U2, Z=0..L);
> dF := diff(U,F);
>
```

$$ro := \mathbf{proc}(Z)\ 15\ \mathbf{end\ proc}$$

$$ri := \mathbf{proc}(Z)\ 14\ \mathbf{end\ proc}$$

$$E := 0.7515000000\ 10^{7}$$

$$L := 2500$$

$$G := 0.2890400000\ 10^{7}$$

$$A := \mathbf{proc}(ro, ri)\ \pi \times (ro(Z)\text{^}2 - ri(Z)\text{^}2)\ \mathbf{end\ proc}$$

$$Ix := \mathbf{proc}(ro, ri)\ 1/4 \times \pi \times (ro(Z)\text{^}4 - ri(Z)\text{^}4)\ \mathbf{end\ proc}$$

$$P = F$$

$$V = F$$

$$M := \mathbf{proc}(Z)\ F \times (Z - L)\ \mathbf{end\ proc}$$

$$U1 := \frac{0.2179821425\ \ 10^{-10}\ F^{2}\ (Z - 2500)^{2}}{\pi}$$

$$U2 := \frac{0.1193009916\ \ 10^{-7}\ F^{2}}{\pi}$$

$$U := 0.03614796813\ F^{2}$$

$$dF := 0.07229593626\ \ F$$

```
>

> # Tapered case # (I)
> # (P) or (F) is the concentrated force at the tip of the mast.
> ro := proc (Z) (5000-Z)/250 end;
> ri := proc (Z) (4750-Z)/250 end;
> E  :=7.515*(10^6);
> L  := 2500;
```

```
> G  :=2.8904*(10^6);
> A := proc (ro,ri) Pi*((ro(Z))^2-(ri(Z))^2) end;
>  Ix := proc (ro,ri) Pi*((ro(Z))^4-(ri(Z))^4)/4 end;
> P=F;
> V=F;
> M := proc (Z) F*(Z-L) end;
> U1 := (M(Z))^2/(2*E*Ix(ro,ri));
> U2 := F^2*(2)/(2*G*A(ro,ri));
> U := int( U1+U2, Z=0..L);
> dF := diff(U,F);
>
```

$$ro := \mathbf{proc}(Z)\ 20 - 1/250 \times Z\ \mathbf{end\ proc}$$

$$ri := \mathbf{proc}(Z)\ 19 - 1/250 \times Z\ \mathbf{end\ proc}$$

$$E := 0.7515000000\ 10^{7}$$

$$L := 2500$$

$$G := 0.2890400000\ 10^{7}$$

$$A := \mathbf{proc}(ro, ri)\ \pi \times (ro(Z)\text{^}2 - ri(Z)\text{^}2)\ \mathbf{end\ proc}$$

$$Ix := \mathbf{proc}(ro, ri)\ 1/4 \times \pi \times (ro(Z)\text{^}4 - ri(Z)\text{^}4)\ \mathbf{end\ proc}$$

$$P = F$$

$$V = F$$

$$M := \mathbf{proc}(Z)\ F \times (Z - L)\ \mathbf{end\ proc}$$

$$U1 := \frac{0.2661343978\ 10^{-6}\ F^2\ (Z-2500)^2}{\pi\left(\left(20-\frac{Z}{250}\right)^4-\left(19-\frac{Z}{250}\right)^4\right)}$$

$$U2 := \frac{0.3459728757\ 10^{-6}\ F^2}{\pi\left(\left(20-\frac{Z}{250}\right)^2-\left(19-\frac{Z}{250}\right)^2\right)}$$

$$U := 0.02473992496\ F^2$$

$$dF := 0.04947984992\ F$$

```
> # Tapered case #(II)
> # (P) or (F) is the concentrated force at the tip of the mast.
> ro := proc (Z) (3125-Z)/125 end;
> ri := proc (Z) (3000-Z)/125 end;
> E  :=7.515*(10^6);
> L  := 2500;
> G  :=2.8904*(10^6);
> A := proc (ro,ri) Pi*((ro(Z))^2-(ri(Z))^2) end;
>  Ix := proc (ro,ri) Pi*((ro(Z))^4-(ri(Z))^4)/4 end;
> P=F;
> V=F;
> M := proc (Z) F*(Z-L) end;
> U1 := (M(Z))^2/(2*E*Ix(ro,ri));
> U2 := F^2*(2)/(2*G*A(ro,ri));
> U := int( U1+U2, Z=0..L);
> dF := diff(U,F);
```

>

$$ro := \mathbf{proc}(Z)\ 25 - 1/125 \times Z\ \mathbf{end\ proc}$$

$$ri := \mathbf{proc}(Z)\ 24 - 1/125 \times Z\ \mathbf{end\ proc}$$

$$E := 0.7515000000\ 10^{7}$$

$$L := 2500$$

$$G := 0.2890400000\ 10^{7}$$

$$A := \mathbf{proc}(ro, ri)\ \pi \times (ro(Z)\text{^}2 - ri(Z)\text{^}2)\ \mathbf{end\ proc}$$

$$Ix := \mathbf{proc}(ro, ri)\ 1/4 \times \pi \times (ro(Z)\text{^}4 - ri(Z)\text{^}4)\ \mathbf{end\ proc}$$

$$P = F$$

$$V = F$$

$$M := \mathbf{proc}(Z)\ F \times (Z - L)\ \mathbf{end\ proc}$$

$$U1 := \frac{0.2661343978\ \ 10^{-6}\ F^{2}\ (Z - 2500\ )^{2}}{\pi \left(\left(25 - \frac{Z}{125} \right)^{4} - \left(24 - \frac{Z}{125} \right)^{4} \right)}$$

$$U2 := \frac{0.3459728757\ \ 10^{-6}\ F^{2}}{\pi \left(\left(25 - \frac{Z}{125} \right)^{2} - \left(24 - \frac{Z}{125} \right)^{2} \right)}$$

$$U := 0.02252226092\ F^{2}$$

$$dF := 0.04504452184\ \ F$$

```
> # Tapered case # (III)
> # (P) or (F) is the concentrated force at the tip of the mast.
> ro := proc (Z) (2692.3076-Z)/96.1538 end;
> ri := proc (Z) (2596.1538-Z)/96.1538 end;
```

```
> E  :=7.515*(10^6);
> L  := 2500;
> G  :=2.8904*(10^6);
> A := proc (ro,ri) Pi*((ro(Z))^2-(ri(Z))^2) end;
>  Ix := proc (ro,ri) Pi*((ro(Z))^4-(ri(Z))^4)/4 end;
> P =F;
> V=F;
> M := proc (Z) F*(Z-L) end;
> U1 := (M(Z))^2/(2*E*Ix(ro,ri));
> U2 := F^2*(2)/(2*G*A(ro,ri));
> U := int( U1+U2, Z=0..L);
> dF := diff(U,F);
>
```

$$ro := \mathbf{proc}(Z)\ 28.00001247 - 0.01040000499 \times Z\ \mathbf{end\ proc}$$

$$ri := \mathbf{proc}(Z)\ 27.00001247 - 0.01040000499 \times Z\ \mathbf{end\ proc}$$

$$E := 0.7515000000\ 10^{7}$$

$$L := 2500$$

$$G := 0.2890400000\ 10^{7}$$

$$A := \mathbf{proc}(ro, ri)\ \pi \times (ro(Z)\text{^}2 - ri(Z)\text{^}2)\ \mathbf{end\ proc}$$

$$Ix := \mathbf{proc}(ro, ri)\ 1/4 \times \pi \times (ro(Z)\text{^}4 - ri(Z)\text{^}4)\ \mathbf{end\ proc}$$

$$P = F$$

$$V = F$$

$$M := \mathbf{proc}(Z)\ F \times (Z - L)\ \mathbf{end\ proc}$$

$U1 :=$

$$\frac{0.2661343978\ 10^{-6}\ F^2\ (Z - 2500)^2}{\pi\ ((28.00001247\ - 0.01040000499\ Z)^4 - (27.00001247\ - 0.01040000499\ Z)^4)}$$

$U2 :=$

$$\frac{0.3459728757\ 10^{-6}\ F^2}{\pi\ ((28.00001247\ - 0.01040000499\ Z)^2 - (27.00001247\ - 0.01040000499\ Z)^2)}$$

$$U := 0.02839615015\ F^2$$

$$dF := 0.05679230030\ F$$

APPENDIX B

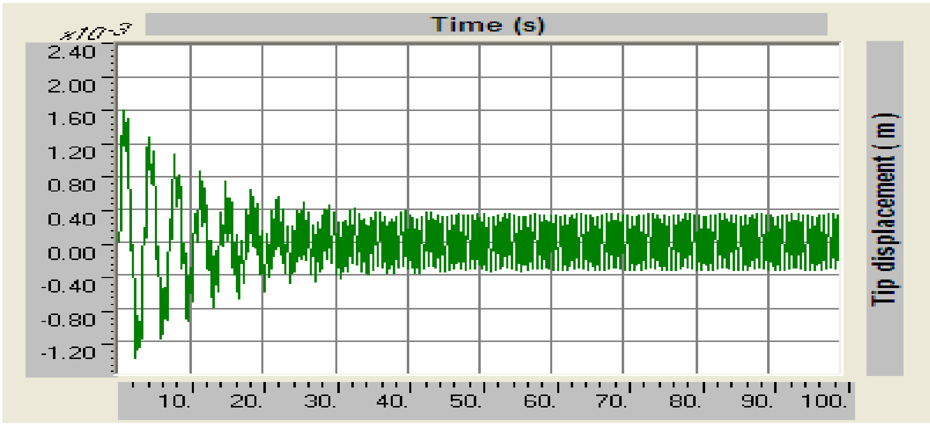

Figure 1 The total dynamic response of the tip of the straight mast, with a wind velocity of 2 m/s and a time span of 100 seconds.

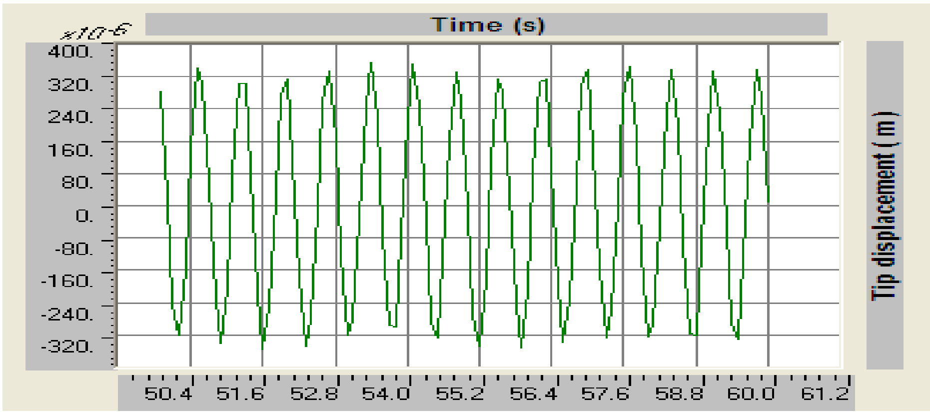

Figure 2 Part of the steady state dynamic response of the tip of the straight mast, with a wind velocity of 2m/s taken between 50 and 60 seconds.

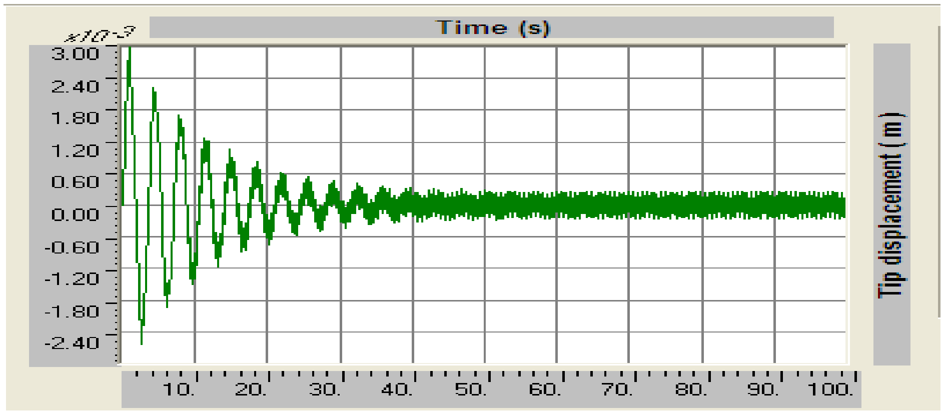

Figure 3 The total dynamic response of the tip of the straight mast, with a wind velocity of 4 m/s and a time span of 100 seconds.

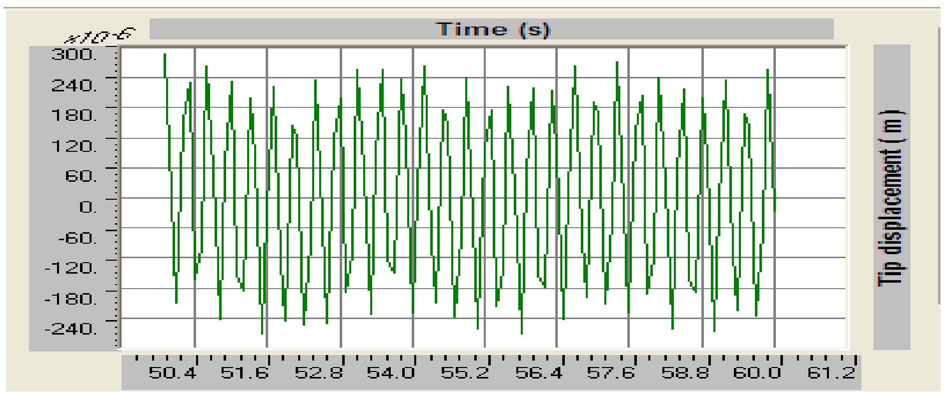

Figure 4 Part of the steady state dynamic response of the tip of the straight mast, with a wind velocity of 4m/s taken between 50 and 60 seconds.

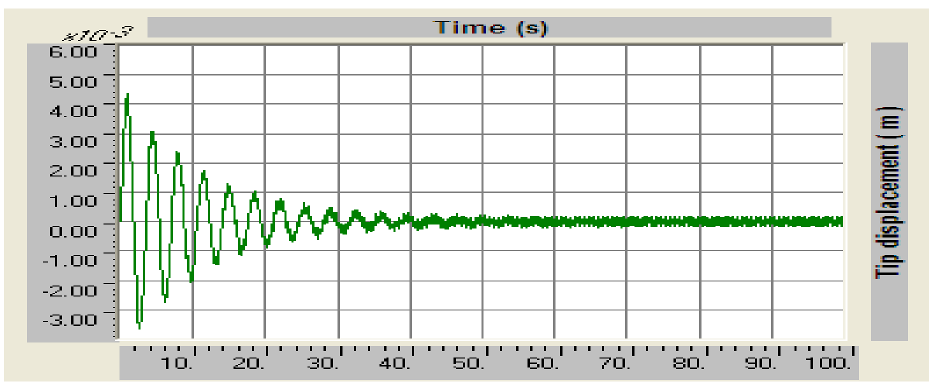

Figure 5 The total dynamic response of the tip of the straight mast, with a wind velocity of 6 m/s and a time span of 100 seconds.

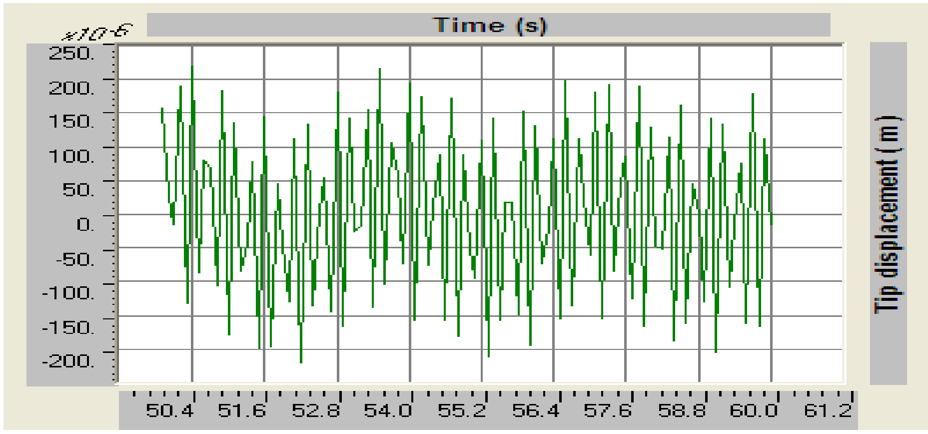

Figure 6 Part of the steady state dynamic response of the tip of the straight mast, with a wind velocity of 6m/s taken between 50 and 60 second.

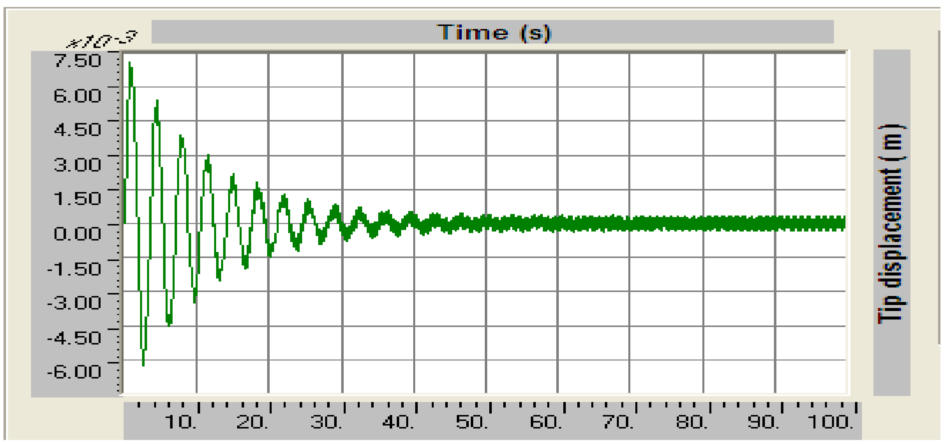

Figure 7 The total dynamic response of the tip of the straight mast, with a wind velocity of 10 m/s and a time span of 100 seconds.

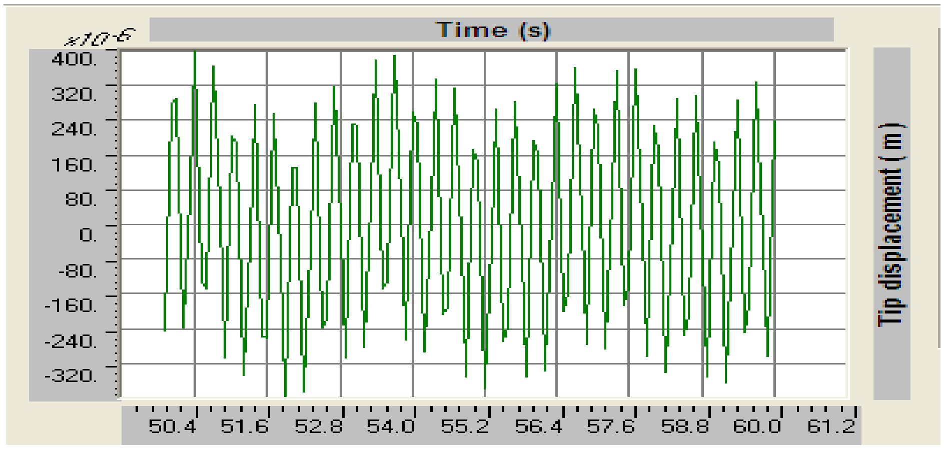

Figure 8 Part of the steady state dynamic response of the tip of the straight mast, with a wind velocity of 10m/s taken between 50 and 60 second.

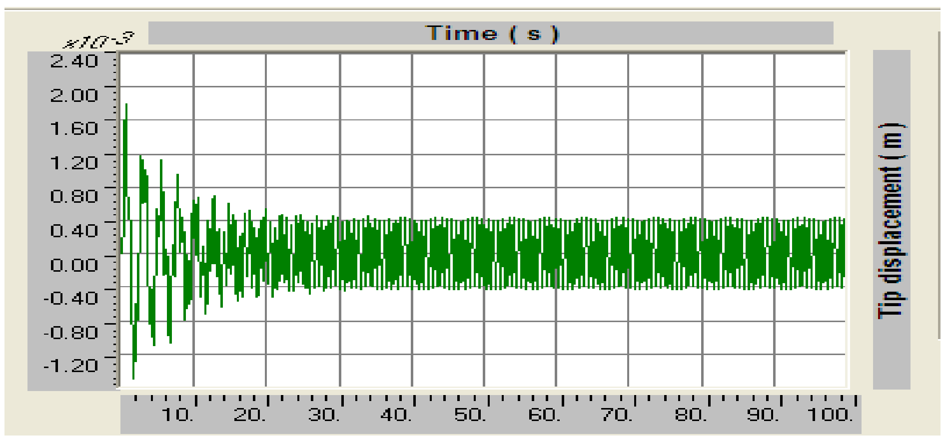

Figure 9 The total dynamic response of the tip of tapered design I, with a wind velocity of 2 m/s and a time span of 100 seconds.

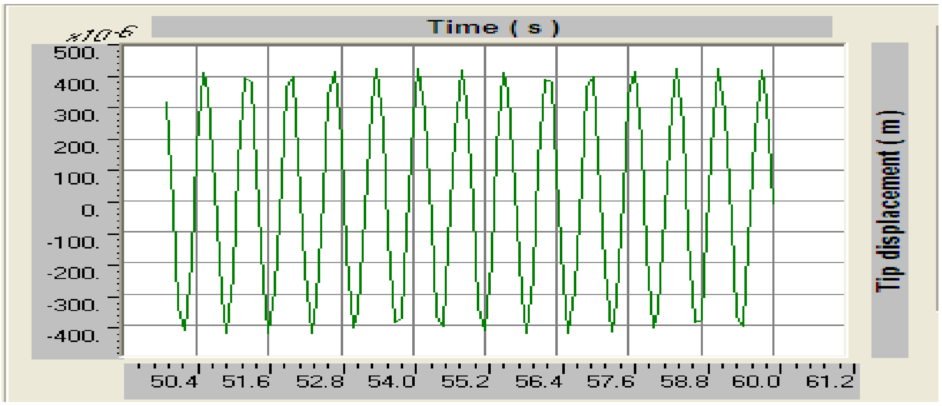

Figure 10 Part of the steady state dynamic response of tapered design I, with a wind velocity of 2m/s taken between 50 and 60 second.

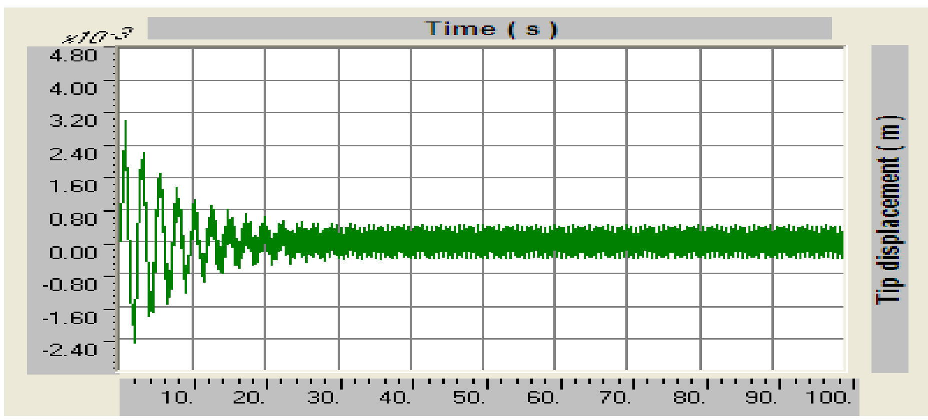

Figure 11 The total dynamic response of the tip of tapered design I, with a wind velocity of 4 m/s and a time span of 100 seconds.

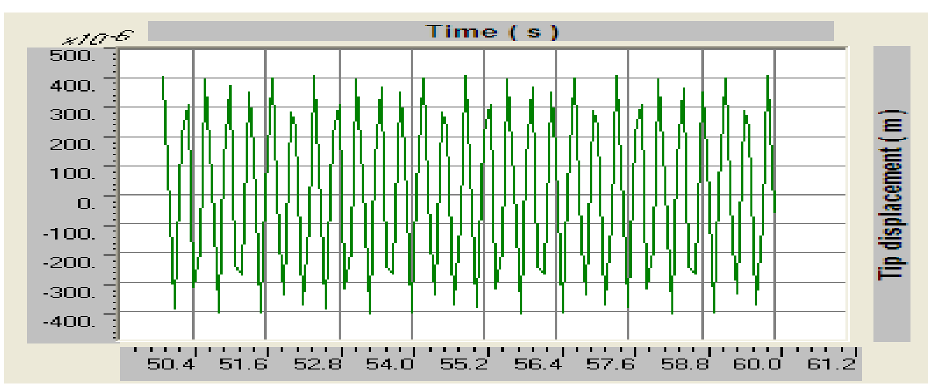

Figure 12 Part of the steady state dynamic response of tapered design I, with a wind velocity of 4 m/s taken between 50 and 60 second.

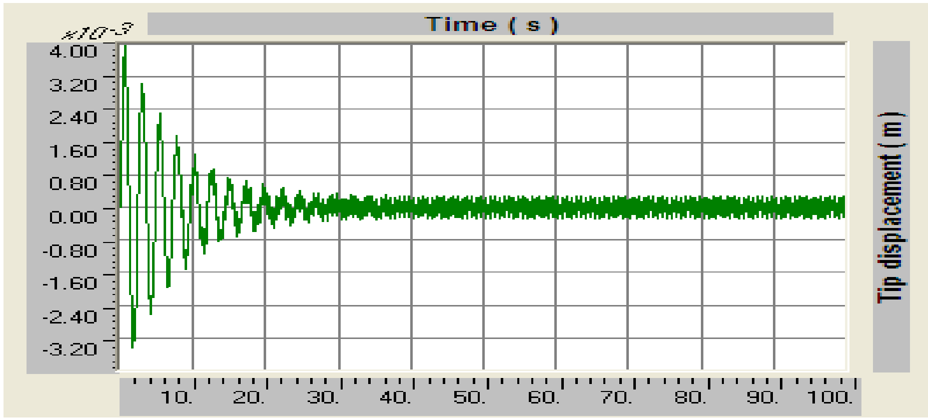

Figure 13 The total dynamic response of the tip of tapered design I, with a wind velocity of 6 m/s and a time span of 100 seconds.

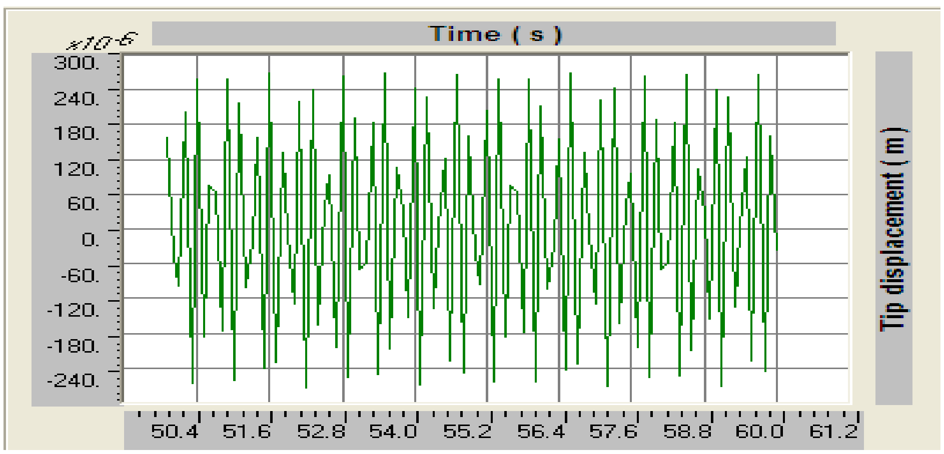

Figure 14 Part of the steady state dynamic response of tapered design I, with a wind velocity of 6 m/s taken between 50 and 60 second.

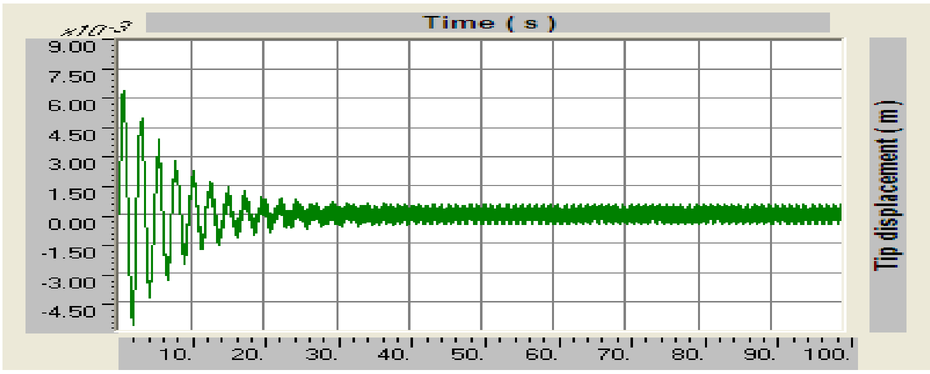

Figure 15 The total dynamic response of the tip of tapered design I, with a wind velocity of 10 m/s and a time span of 100 seconds.

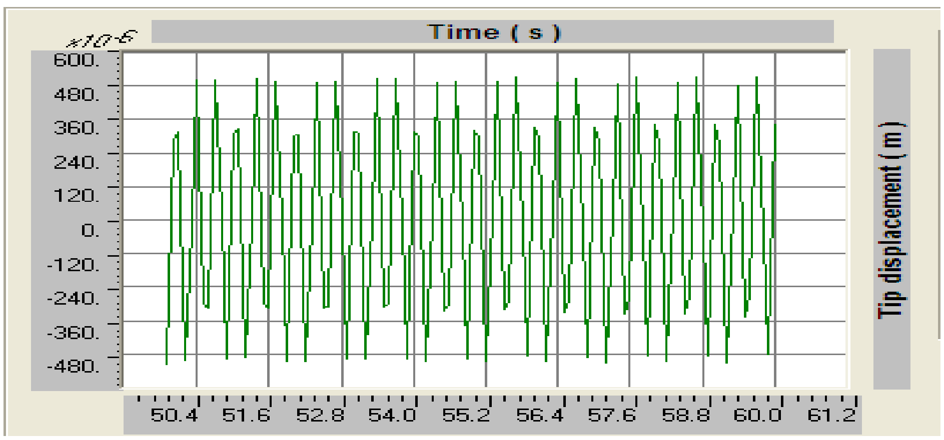

Figure 16 Part of the steady state dynamic response of tapered design I, with a wind velocity of 10 m/s taken between 50 and 60 second.

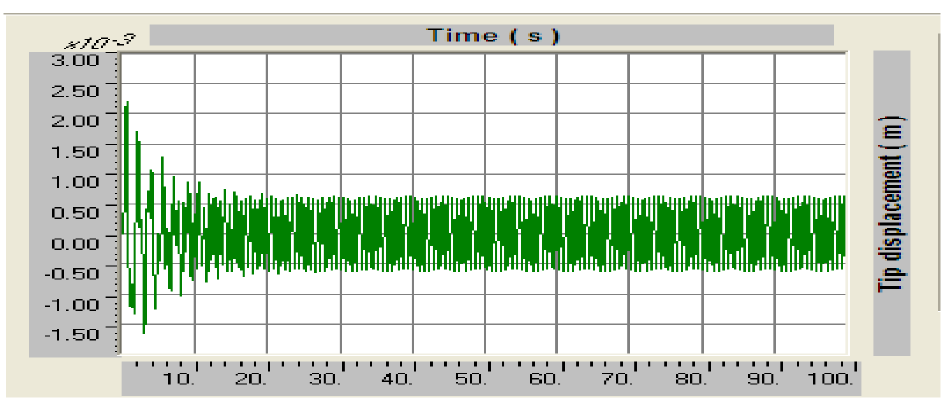

Figure 17 The total dynamic response of the tip of tapered design II, with a wind velocity of 2 m/s and a time span of 100 seconds.

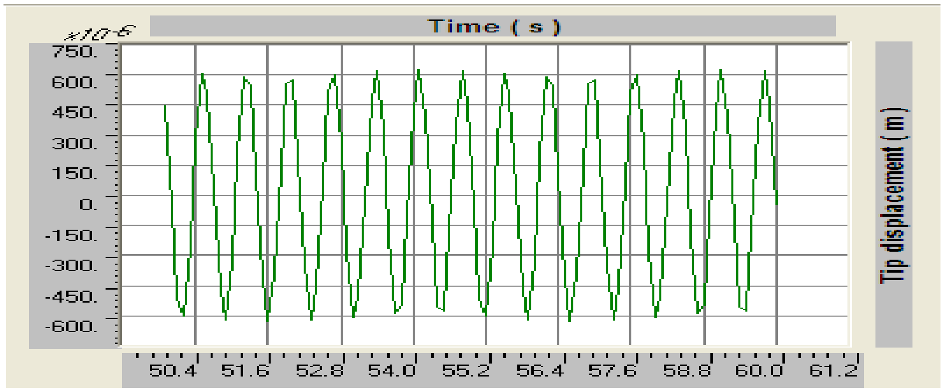

Figure 18 Part of the steady state dynamic response of tapered design II, with a wind velocity of 2 m/s taken between 50 and 60 second.

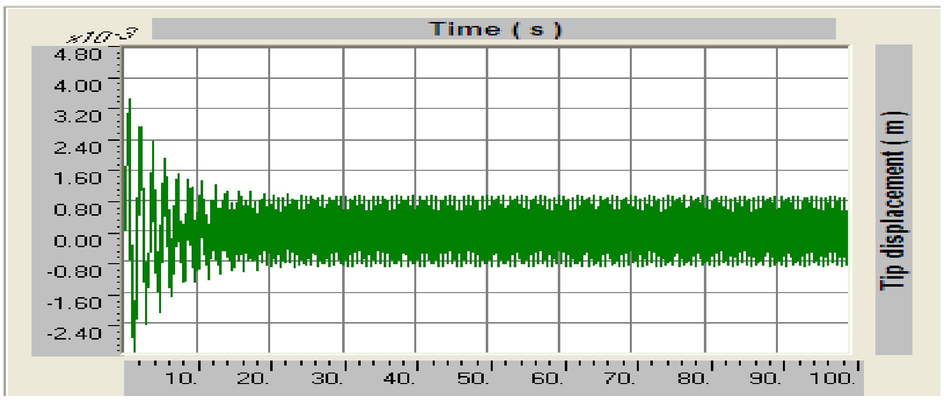

Figure 19 The total dynamic response of the tip of tapered design II, with a wind velocity of 4 m/s and a time span of 100 seconds.

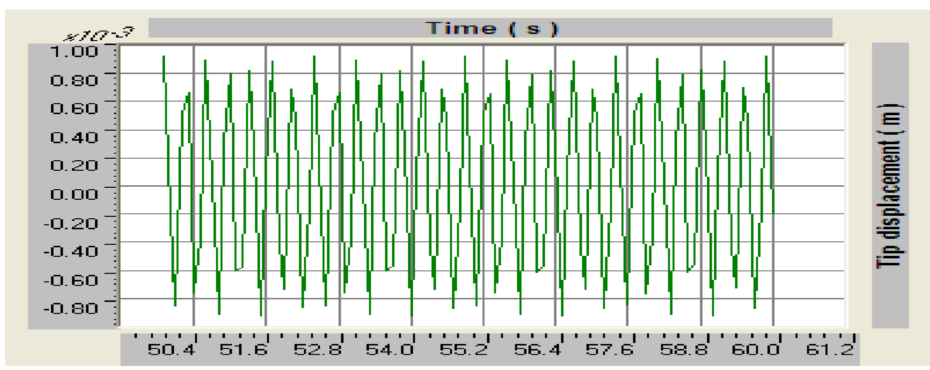

Figure 20 Part of the steady state dynamic response of tapered design II, with a wind velocity of 4 m/s taken between 50 and 60 second.

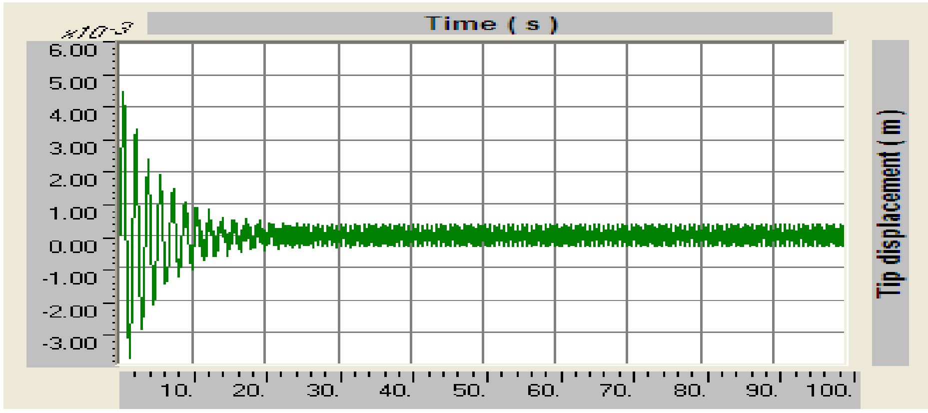

Figure 21 The total dynamic response of the tip of tapered design II, with a wind velocity of 6 m/s and a time span of 100 seconds.

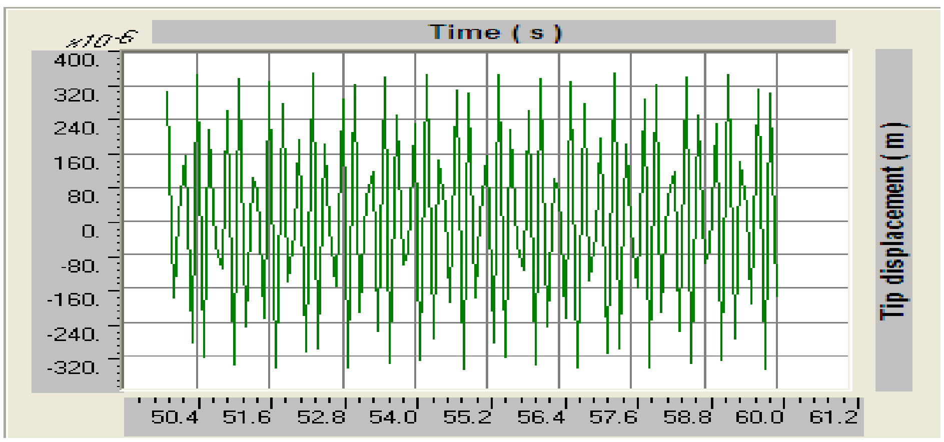

Figure 22 Part of the steady state dynamic response of tapered design II, with a wind velocity of 6 m/s taken between 50 and 60 second.

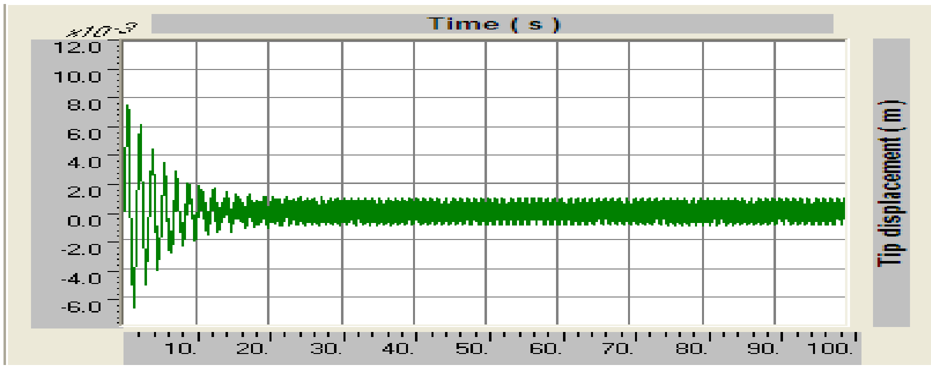

Figure 23 The total dynamic response of the tip of tapered design II, with a wind velocity of 10 m/s and a time span of 100 seconds.

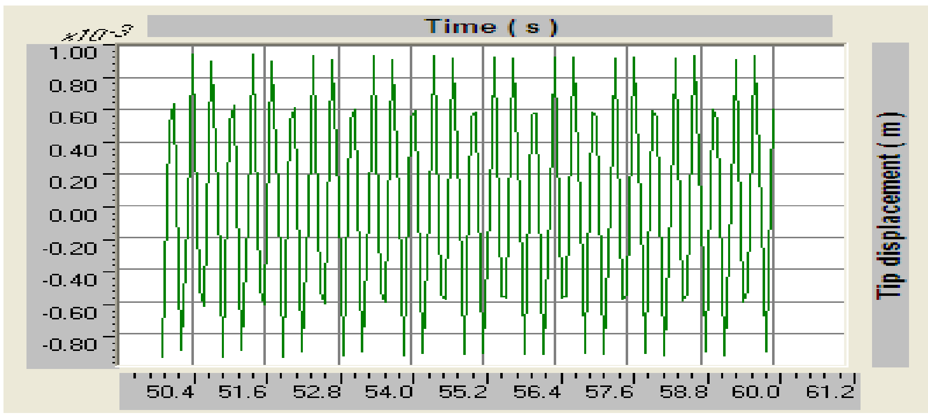

Figure 24 Part of the steady state dynamic response of tapered design II, with a wind velocity of 10 m/s taken between 50 and 60 second.

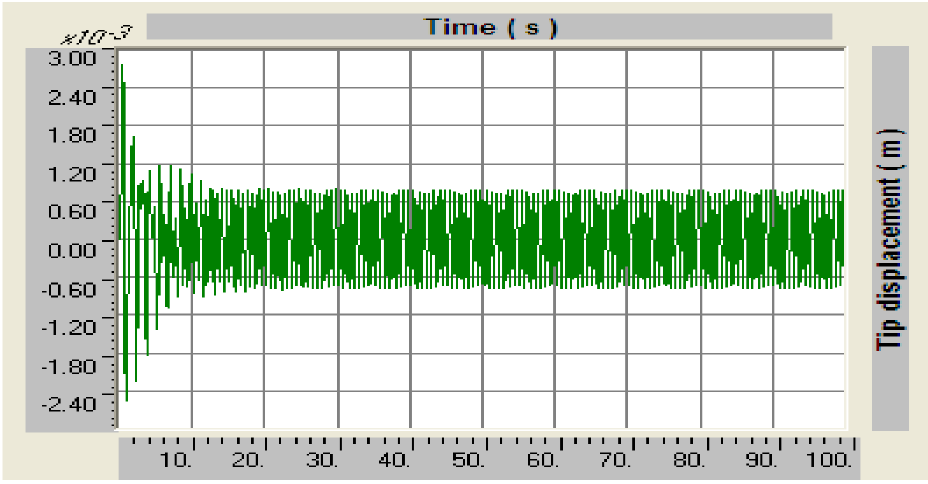

Figure 25 The total dynamic response of the tip of tapered design III, with a wind velocity of 2 m/s and a time span of 100 seconds.

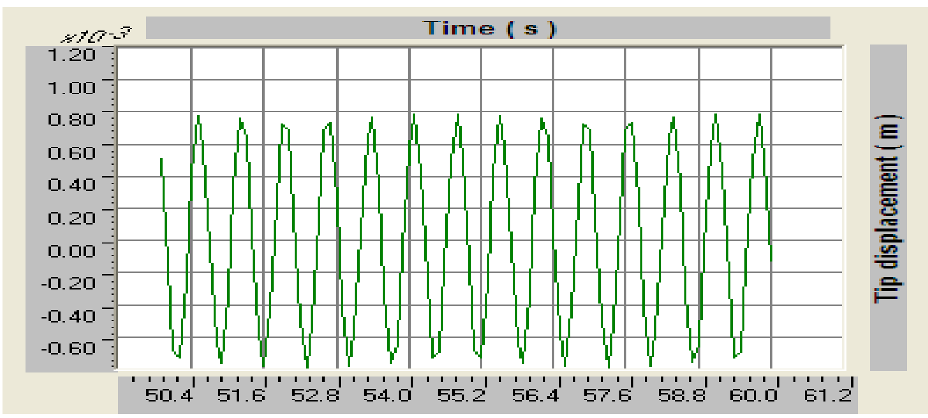

Figure 26 Part of the steady state dynamic response of tapered design III, with a wind velocity of 2 m/s taken between 50 and 60 second.

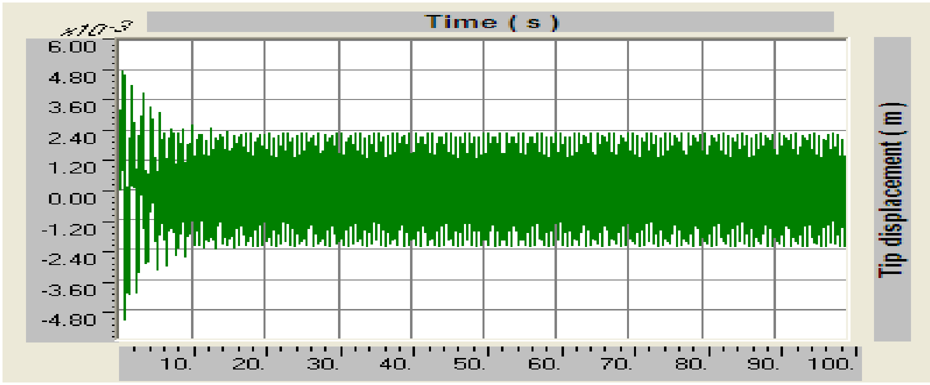

Figure 27 The total dynamic response of the tip of tapered design III, with a wind velocity of 4 m/s and a time span of 100 seconds.

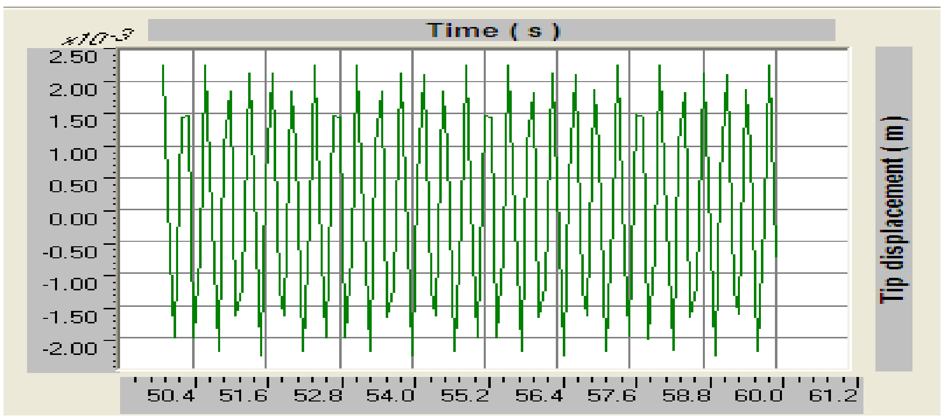

Figure 28 Part of the steady state dynamic response of tapered design III, with a wind velocity of 4 m/s taken between 50 and 60 second.

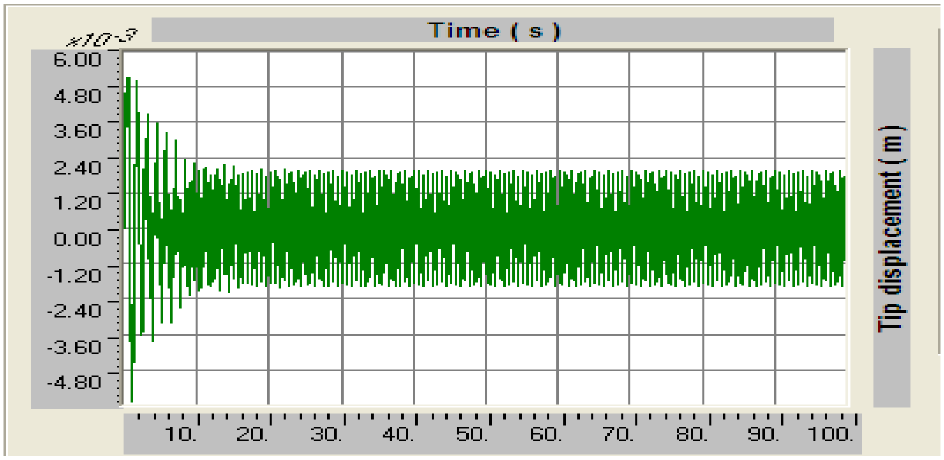

Figure 29 The total dynamic response of the tip of tapered design III, with a wind velocity of 6 m/s and a time span of 100 seconds.

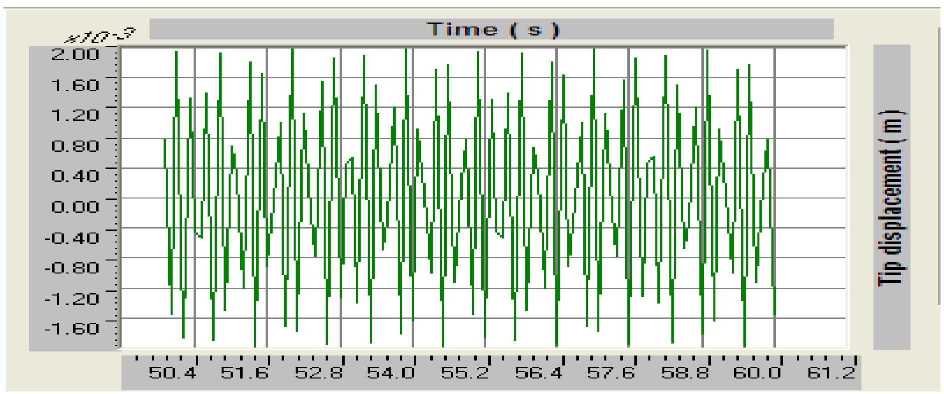

Figure 30 Part of the steady state dynamic response of tapered design III, with a wind velocity of 6 m/s taken between 50 and 60 second.

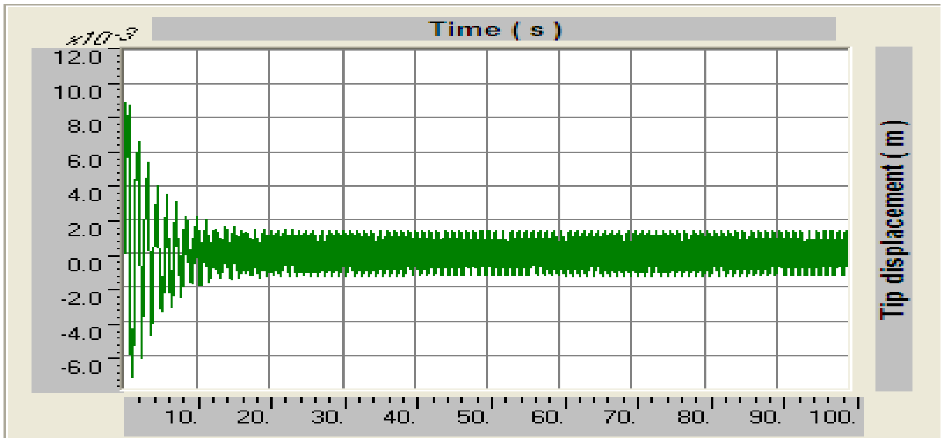

Figure 31 The total dynamic response of the tip of tapered design III, with a wind velocity of 10 m/s and a time span of 100 seconds.

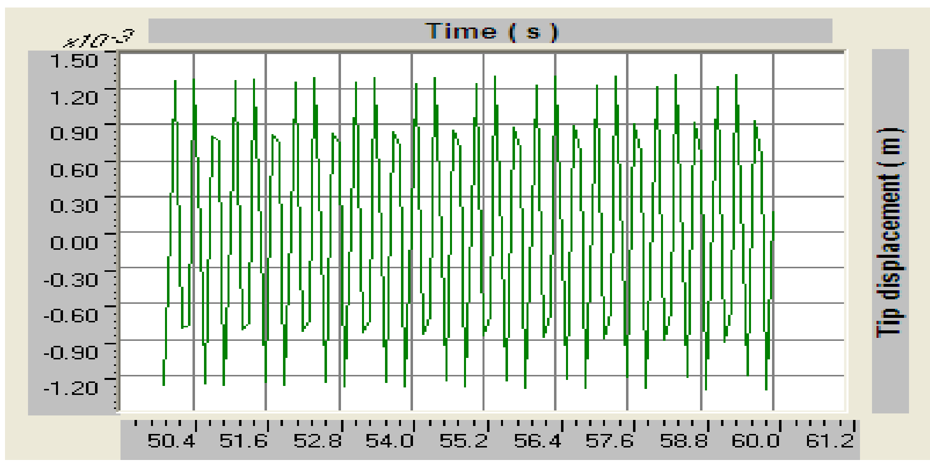

Figure 32 Part of the steady state dynamic response of tapered design III, with a wind velocity of 10 m/s taken between 50 and 60 second.